U N I V E R S I T Y   O F   R E A D I N G

A Practical Model For Heavy Oil Recovery

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D E P A R T M E N T   O F   M A T H E M A T I C S

Abstract

A simple model is developed to describe the production of heavy oil following the injection of steam into the reservoir. A numerical algorithm applicable to the model is described and some results presented.

Acknowledgements

It is with pleasure that I acknowledge the help and support given to me by Dr. M.J. Baines and Mr. D.E. Heath throughout this work.

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CONTENTS

Introduction

The problem under consideration is the optimization of heavy oil recovery by cyclic steam injection. The first part of this introduction provides some background information on oil recovery while the second part goes on to describe how the problem is tackled within the report.

Oils can be divided into two categories, light oils and heavy oils. Light oils have a low viscosity while heavy oils have a high viscosity [10]. The viscosity of a fluid is a measure of how easily that fluid will flow, for example, water has a very low viscosity while treacle has a high viscosity.

Light oils can be extracted by primary and secondary recovery techniques which involve allowing the fluid to flow out under the natural pressure of its surrounds [9]. These methods cannot be applied to the extraction of heavy oils, whose viscosity is far too high for such methods to be effective; their viscosity needs to be reduced. This is achieved by various thermal stimulation techniques [11] which raise the temperature of the oil, effectively reducing its viscosity. The most widely used of these methods is that of cyclic steam injection [10].

Cyclic steam injection, or "huff and puff" as it is also known, involves firstly injecting steam for a period of time into the reservoir in order to create a steam zone. This is followed by a soak period, during which the reservoir is left so that the heat injected distributes itself throughout the steam zone to create a heated zone in which the viscosity of the oil has been reduced (see fig. 1).

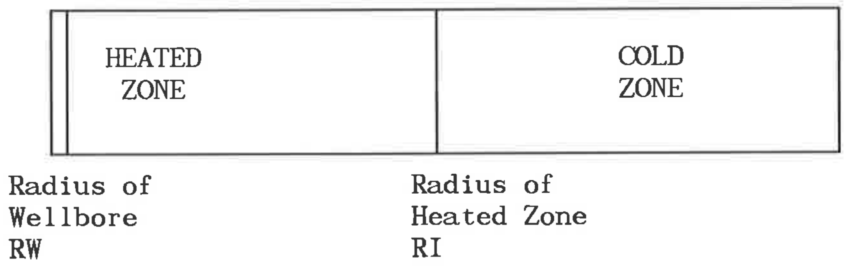


FIG. 1

The final stage is the production period which together with the injection and soak period, form one cycle of the process. This cycle is then repeated successively until it becomes uneconomic to continue to do so.

The eventual aim of the work reported here is to maximise the overall profit from the process, taking into account such factors as running costs of the well, the cost of injecting steam and the cost of extracting the oil and water produced.

Before any optimization can be considered, we need to develop a mathematical model for the injection, soak and production periods. There is some controversy surrounding the soak period and its benefits, and so for the purposes of this exercise the soak period is neglected in that a mathematical model is not developed for it: the effects of a soak period are however assumed to have taken place in order to provide the initial conditions for the production period.

It is a relatively simple exercise to develop a mathematical model of the injection period [5] and there are a number of models for the production period already available [2], [6]-[8]. The subject of this report is the development of a very simple model for the production period that may be used later in an optimisation routine.

In the first section of the report the assumptions that have been made during the development of the model are presented. The main interest during the production period is in the production rate of oil and how it varies with time, and the derivation of this quantity is the subject of section two. In order to calculate the production rate, expressions for other variables are needed and these are presented in sections 2-5. Finally, in section 6, the numerical algorithm for the model is described together with some of the results obtained.

1. Assumptions

When developing a mathematical model of a physical process it is rare for all of the characteristics of the process to be incorporated into the model, as doing so would normally produce a model that was far too complicated to work with easily or efficiently. It is therefore usual to make some assumptions about the physical process in order to simplify the mathematical model and make it workable. Listed below are some of those that are generally accepted for reservoir modelling [2].

A1. The reservoir has a constant thickness and is homogeneous and isotropic.

A2. The reservoir is initially saturated with oil and water.

A3. A pseudo-steady state flow is assumed within the reservoir.

In addition to the above assumptions the following have also been made in order to simplify the modelling process even further.

A4. After the injection period the steam occupies a cylindrical volume, not conical [2], which is referred to as the heated zone.

A5. The oil mobilized by the steam is contained within the cylindrical heated zone, not in a strip at the edge of the heated zone, [2], [6] and [8].

A6. The potential that causes the water and oil to flow into the well is a pressure drop which is taken to be the difference between the steam pressure at the temperature in the heated zone and the pressure at the well-bore, the effects of gravity are ignored.

A7. The flow rates for oil and water are constant throughout the heated zone at any given time.

A8. There is no flow across the front between the heated and cold zones.

A9. There is no heat loss to the over/under burden during the production period.

These assumptions enable a relatively simple mathematical model to be developed for the production period, modelling quantities such as the flow rate of oil, the change in temperature and the saturations of mobile oil and water throughout the production period. The derivation of expressions for such quantities are described in the next four sections.

2. Derivation Of The Production Rate

The derivation of the production rate used here is based upon the radial diffusivity equation, the basic partial differential equation for the radial flow of any single phase fluid in a porous medium, and the definition of compressibility [1].

The diffusivity equation in radial form is

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] = \frac{\phi}{}\frac{\mu c}{k}\frac{\partial p}{\partial t} \qquad (2.1)$$

where

r = radius

p = pressure

ϕ = porosity

μ = viscosity

c = compressibility

k = effective permeability.

Assumption (A3), that of pseudo-steady state, implies that $\frac{\partial p}{\partial t}$ is equal to some constant. This constant can be found from the definition of compressibility, i.e. at each point

$$cV\frac{dp}{dt} = -\frac{dV}{dt} = q(t) \qquad (2.2)$$

where

V = volume

q = production (flow) rate.

Consider producing oil/water from the whole of the heated zone. The effective volume from which oil/water can be produced is (see fig. 1)

$$V = \pi(RI^2 - RW^2)h\phi S(t) \qquad (2.3)$$

where

S = saturation.

Equation (2.3) can be substituted into equation (2.2) to give

$$c\pi(RI^2 - RW^2)h\phi S(t)\,\frac{\partial p}{\partial t} = -\,q(t)$$

i.e.

$$\frac{\partial p}{\partial t} = \frac{-\,q(t)}{c\pi(RI^2-RW^2)h\phi S(t)}\,. \tag{2.4}$$

Hence the original p.d.e. (2.1) can be written

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial p}{\partial r}\right] = \frac{-\,\mu}{k(t)\pi(RI^2-RW^2)hS(t)}\,q(t)\,. \tag{2.5}$$

Multiplying both sides of (2.5) by r and integrating with respect to r gives

$$\left[r\,\frac{\partial p}{\partial r}\right]_r^{RI} = \frac{-\,\mu}{k(t)\pi(RI^2-RW^2)hS(t)}\,q(t)\left[\frac{r^2}{2}\right]_r^{RI}. \tag{2.6}$$

The assumption (A8), that of no flow at the hot/cold front, provides the boundary condition $\frac{\partial p}{\partial r} = 0$ at $r = RI$ and so (2.6) can be evaluated and rearranged to give

$$\frac{\partial p}{\partial r} = \frac{\mu}{2k(t)\pi(RI^2-RW^2)hS(t)}\,q(t)\left[\frac{RI^2}{r} - r\right]. \tag{2.7}$$

Integrating (2.7) with respect to r and writing $\Delta p(t) = p(RI,t) - p(RW,t)$ results in the following expression for $\Delta p(t)$,

$$\Delta p(t) = \frac{\mu}{2k(t)\pi(RI^2-RW^2)hS(t)}\,q(t)\left[RI^2\ln(RI/RW) - 0.5(RI^2-RW^2)\right]. \tag{2.8}$$

Neglecting terms of order $\left[\frac{RW}{RI}\right]^2$ and rearranging equation (2.8) gives

$$q(t) = \frac{2\pi hk(t)S(t)\,\Delta p(t)}{\mu[\ln(RI/RW) - 0.5]}, \tag{2.9}$$

the final expression for the production (flow) rate of oil/water at time t used in this report. In the next section the method of calculating the temperature during the production period is described.

3. <u>Calculating Temperature</u>

It is very important to be able to calculate the temperature in the heated zone during the production period because as the temperature changes so does the rate at which oil and water are being produced. As the production period progresses the temperature of the oil remaining in the heated zone will decrease and so its viscosity will increase, thus reducing the rate at which the oil is being produced.

In order to calculate the temperature during the production period a heat balance is carried out involving the oil and water in the heated zone at the beginning of the timestep, and the oil and water that has been produced during the timestep together with that remaining in the heated zone of the reservoir.

Consider the heated zone at the beginning of a timestep during the production period.

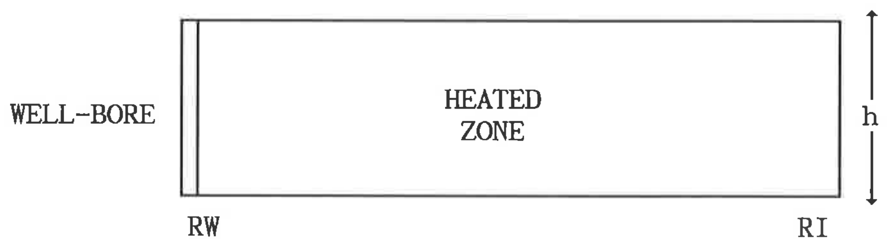


FIG. 2

The volume of oil contained in the heated zone is $\pi(RI^2-RW^2)h\phi S_o)$, and similarly the volume of water contained in the heated zone is $\pi(RI^2-RW^2)h\phi S_w$. The heat contained within the oil and water in the heated zone at the beginning of the production timestep, H_B, can therefore be expressed as

$$H_B = \pi(RI^2-RW^2)h\phi[S_o(T-T_R)M_o + S_w(T-T_R)M_w] \quad (3.1)$$

where

$M_{o,w}$ = heat capacity of oil/water

T_R = original temperature of the reservoir

T = temperature of the heated zone at the beginning of the timestep.

In order to simplify the heat balance process the change over a very short period of time, Δt, in the saturation and heat capacity of the oil and water in the heated zone is neglected for the purpose of this calculation only. The heat contained at the end of the timestep in the oil and water remaining in the heated zone, H_A, can be expressed as

$$H_A = \pi(RI^2-RW^2)h\phi[S_o(T+\Delta T-T_R)M_o + S_w(T+\Delta T-T_R)M_w] \ . \quad (3.2)$$

The heat that has been removed from the heated zone due to the production of oil and water, H_P, can be expressed as

$$H_P = q_w(t)\Delta t(T-T_R)M_w + q_o(t)\Delta t(T-T_R)M_o \ . \quad (3.3)$$

The heat balance equation is

$$H_B - H_A = H_P \quad (3.4)$$

making the assumption that no heat is lost to the surrounding rock (A9). Substituting equations (3.1), (3.2) and (3.3) into (3.4) the following equation is obtained:

$$\pi(RI^2-RW^2)h\phi M_o S_o[(T-T_R) - (T+\Delta T)]$$
$$+ \pi(RI^2-RW^2)h\phi M_w S_w[(T-T_R) - (T+\Delta T-T_R)]$$
$$= q_w(t)\Delta t(T-T_R)M_w + q_o(t)\Delta t(T-T_R)M_o \ . \quad (3.5)$$

Rearranging equation (3.5) gives

$$- \pi(RI^2-RW^2)h\phi[M_oS_o+M_wS_w]\Delta t$$

$$= [M_wq_w(t) + M_oq_o(t)](T-T_R)\Delta t \ . \tag{3.6}$$

Allowing $\Delta t \to 0$, (3.6) can be expressed as the differential equation

$$w \frac{dT}{dt} = - (T-T_R)h(t) \tag{3.7}$$

where, remembering that $P_{o,w}$, $S_{o,w}$ and $M_{o,w}$ are evaluated at t for the purpose of this calculation,

$$h(t) = \frac{M_w(t)q_w(t) + M_o(t)q_o(t)}{M_w(t)S_w(t) + M_o(t)S_o(t)}$$

and

$$w = \pi(RI^2-RW^2)h\phi \ .$$

Rearranging (3.7) gives

$$w\int_{T_s}^{T} \frac{d\hat{T}}{\hat{T} - T_R} = - \int_0^t h(\tau)d\tau \tag{3.8}$$

where

T_s = temperature of the heated zone at the beginning of the production period.

Integrating (3.8) results in

$$w \ln\left[\frac{T - T_R}{T_S - T_R}\right] = - Q(t) \tag{3.9}$$

where

$$Q(t) = \int_0^t h(\tau)d\tau \ , \tag{3.10}$$

$Q(t)$ can be thought of as the average amount of fluid produced from unit volume of the heated zone of the reservoir during the production timestep under consideration. Equation (3.9) can be rearranged to obtain

$$T = T_R + (T_S - T_R)\exp(-\frac{1}{w} Q(t)) , \qquad (3.11)$$

the final expression for temperature at time t, from which it can be seen that the temperature decreases exponentially with time. The next section describes how the expression for temperature is used to calculate various other quantities.

4. Quantities Dependent On Temperature

The expression obtained for the production (flow) rate of oil/water in section two depends on the variable quantities saturation, effective permeability and dynamic viscosity of oil/water and the pressure drop across the heated zone. Although all of these quantities can ultimately be expressed as a function of time during the production period, it is more usual to think of some of them as functions of temperature, which is why the temperature is recalculated at the end of each timestep.

The quantities that are to be treated as functions of temperature are pressure and dynamic viscosity. The easiest of these to deal with is the pressure drop Δp, defined by $\Delta p = p(RI,t) - p(RW,t)$ where $p(RW,t)$ is known and $p(RI,t) = p_s$, the pressure of steam at temperature T_s. The correlation

$$p_s = \left[\frac{T_s}{115.95}\right]^{4.4543} \text{ psia} \qquad T_s \text{ in } {}^{o}\text{F} \tag{4.1}$$

is often used [2].

The dynamic viscosity is not so easily expressed as a function of temperature. The dynamic viscosity of water provides no difficulty and the following expression is generally used [2],

$$\mu_w = 0.66\left[\frac{T}{100}\right]^{-1.14} \text{ cp} \qquad T \text{ in } {}^{o}\text{F}, \tag{4.2}$$

but the expression for μ_o is more complicated. Two separate expressions for ρ_o and v_o (where v_o is the kinematic viscosity of oil) are used, which are substituted into the relationship $\mu_o = \rho_o v_o$. The standard correlation for ρ_o is [2]

$$\rho_o = \rho_{ostd} - 0.02184(T - T_{ostd}) \quad lb/ft^3 \tag{4.3}$$

where T in oF and T_{ostd} and ρ_{ostd} are standard temperature and density respectively for oil while the expression for v_o is [3],

$$v_o = (v_1 + 0.8)^{(T_1/T)^{\beta}} - 0.8 \quad cSt \tag{4.4}$$

where

$$\beta = \frac{1}{\log(T_2/T_1)} * \log\left[\frac{\log(v_1 + 0.8)}{\log(v_2 + 0.8)}\right]$$

v_1 = kinematic viscosity (cSt) of oil at T_1

v_2 = kinematic viscosity (cSt) of oil at T_2.

The above correlations allow the production (flow) rates of oil and water to be calculated during a specific timestep provided that the temperature of the heated zone is known for that timestep.

In order to calculate the temperature of the heated zone for a new timestep the volumetric heat capacities and saturations of oil and water have to be evaluated using information from the previous timestep. In particular heat capacity is a function of temperature and the following relationships are used [2],

$$M_o = (3.065 + 0.00355T)\sqrt{\rho_o} \quad T \text{ in } oF \tag{4.5}$$

where ρ_o is expressed by (4.3) and

$$M_w = \rho_w C_w \tag{4.6}$$

where C_w is the specific heat capacity of water and is expressed [2] by

$$C_w = \frac{68}{(T-T_R)}\left[\left(\frac{T}{100}\right)^{1.24} - \left(\frac{T_R}{100}\right)^{1.24}\right] \quad \text{Btu/lb - °F} \qquad (4.7)$$

and ρ_w is expressed [2] by

$$\rho_w = 62.4 - 11 \ln\left|\frac{705 - T_{wstd}}{705 - T}\right| \quad \text{lb/ft}^3 \qquad (4.8)$$

where T in °F and T_{wstd} is the standard temperature for water.

In this section expressions have been given for those quantities which are treated as functions of temperature, but this still leaves expressions for permeabilities and saturations to be provided.

5. Permeability and Saturation

It is important that expressions for the relative permeability to, and saturation of, oil and water are available as they are required both directly and indirectly for the calculation of production (flow) rates, as shown in sections two and four.

The relative permeabilities can be treated as simple functions of saturation where the saturations have to be calculated from the amount of water that has been produced so far during the production period. First of all consider the evaluation of relative permeabilities [4].

In order to calculate the relative permeabilities a normalized water saturation, S_w^*, is calculated,

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{orw}} \tag{5.1}$$

where

S_w = water saturation

S_{wi} = initial water saturation

S_{orw} = residual oil saturation in the presence of water.

The relative permeability to water, K_w, is given by

$$K_w = K_{end} \times S_w^* \tag{5.2}$$

where K_{end} is the relative permeability to water at

$$(1 - S_{orw}) ;$$

the relative permeability to oil, K_o, is given by

$$K_o = 1 - \frac{K_w}{K_{end}} = 1 - S_w^* . \tag{5.3}$$

All that are required now are the expressions for the saturation of oil and water; as mentioned earlier, these depend upon the amount of water that has been produced so far in the production period [2].

Before production starts it is assumed that the only mobile fluid in the heated zone is water, so that during production the oil saturation increases. The saturation of water is calculated from the expression,

$$S_w = \overline{S_w} - (\overline{S_w} - S_{wi}) \frac{W_p}{WIP} \tag{5.4}$$

where

$\overline{S_w}$ = $1 - S_{orw}$, the saturation of water at the beginning of the production period,

W_p = volume of water produced so far

WIP = volume of water in place in the reservoir at the beginning of the production period.

It is then simple to calculate the saturation of mobile oil,

$$S_o = 1 - S_{orw} - S_w . \tag{5.5}$$

The correlations described in this section and the previous one together with the expressions for temperature and production (flow) rates in sections three and two enable a numerical model of the production period to be developed.

6. Numerical Solution of Model and Results

Once the mathematical model has been developed for the production period the next stage is to find a numerical algorithm to apply to the model.

The initial conditions and part of the data required for the numerical solution has to be supplied by the state of the reservoir at the end of the injection period. The information required is the radius and temperature of the heated zone together with the volume of water in place at the end of the injection period.

The volume of water in place is the amount of (cold water equivalent) steam injected plus the original amount of water in the heated zone,

$$\text{WIP} = \text{water injected} + \text{water originally in reservoir.}$$

The WIP is needed to calculate the saturation of water during the production period. There is no calculation needed to find the radius and temperature of the heated zone after the injection period, since these quantities are known.

It is the production rate of oil that is of most interest during the production period but, in order to calculate this, other quantities such as saturation, relative permeability and temperature have to be evaluated. It would be far too complicated, (see section two), to express the production rate of oil as an explicit function of time using the expressions for saturation, relative permeability and pressure described in sections four and five. Therefore the production period is divided up into timesteps and quantities treated as constant over each timestep, thus making the production rate into a step function.

An algorithm then has to be developed which dictates the order in which expressions are evaluated.

The algorithm requires integrals to be evaluated of the form

$$\int_0^{t_i} h(\tau)d\tau \quad \text{(see (3.8))} \quad \text{and} \quad \int_0^{t_i} q_w(\tau)d\tau$$

(see (5.4)), which creates a problem since, at the beginning of the i^{th} timestep, $h(t_i)$ and $q_w(t_i)$ are unknown quantities; therefore a predictor-corrector technique is used.

A numerical quadrature of the form

$$\int_0^{t_i} h(\tau)d\tau \approx \sum_{k=0}^{i=1} \alpha_k h(t_k) + \Delta t h(t_{i-1})$$

is used to predict values for the integrals, which allows the algorithm to be executed once to predict values for the unknowns such as q_w and ρ_w. The algorithm is then repeated but using the predicted values as the new starting values. On the second time round a numerical quadrature of the form

$$\int_0^{t_i} h(\tau)d\tau \approx \sum_{k=0}^{i} \alpha_k h(t_k)$$

can be used.

The algorithm used is shown in fig. 3, remembering that properties of the reservoir such as height and porosity together with information from the injection period are known. The flow chart is constructed in such a way that the dependence of quantities on other quantities is easily seen.

The information available at the end of the injection period together with the data for reservoir properties and the algorithm provides a numerical model of the production period.

The algorithm (see fig. 3) was implemented to obtain the results shown in graphs 2-8, which demonstrate how quantities change during the production period. The length of the injection period chosen was five days and the corresponding radius of the steam zone can be found in table 1. Graph 1 demonstrates the development of the steam zone for an injection period of up to eight days.

It should be noted that the length of the production period corresponding to a certain injection time has an upper limit, since there is only so much oil that can be produced. The maximum lengths of production periods for various injection times are given in table 1 and the oil production profiles up to the maximum times shown in graphs 9-16.

The times and values of the maximum production rates are summarised in table 3 from which it can be seen that as the injection period increases the maximum rate of oil production decreases and occurs later. This may well affect the optimum injection and production times for maximum profit when the complete cyclic process is considered.

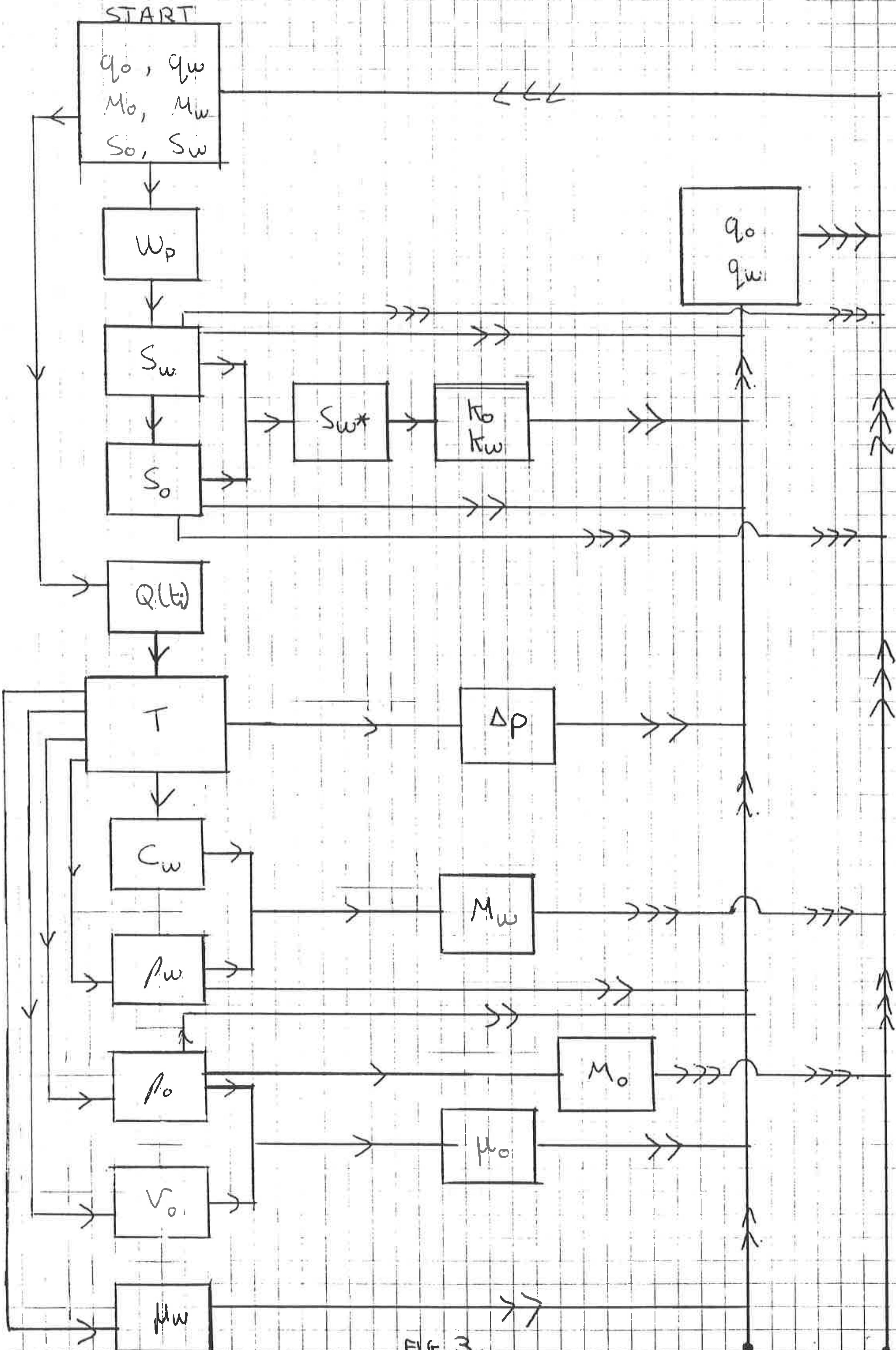

START
qo, qw
Mo, Mw
So, Sw
Wp
Sw
So
Sw*
Ko
Kw
qo
qw
Q(ti)
T
Δp
Cw
ρw
Mw
ρo
Mo
μo
Vo
μw

FIG. 3.

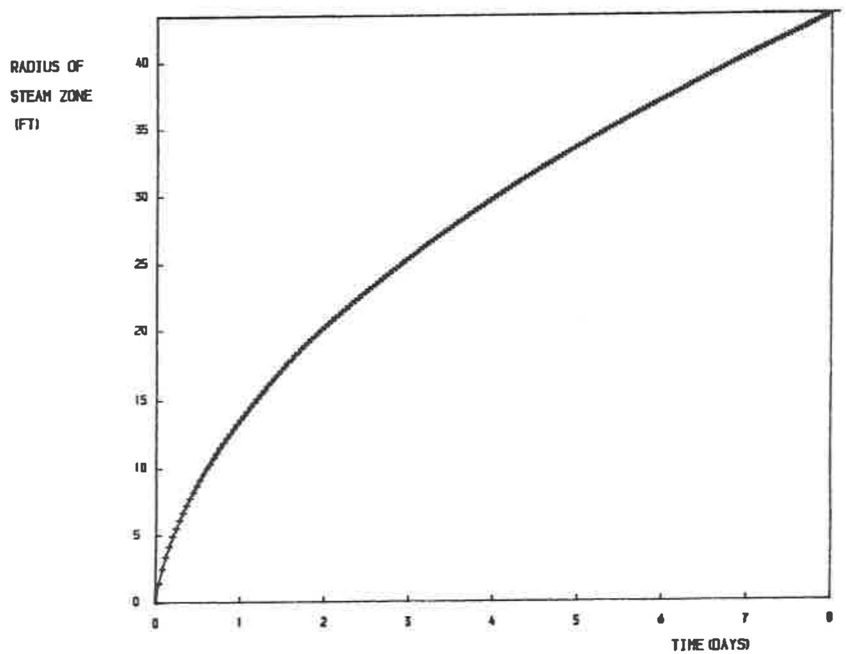
Length of injection period (days).	Rate of steam injection (lbs/day).	Radius of steam zone (ft).	Maximum length of production period (days).
1	500	13.63	36
2	500	20.32	90
3	500	25.49	150
4	500	29.85	215
5	500	33.70	282
6	500	37.18	351
7	500	40.38	422
8	500	43.36	495

TABLE 1

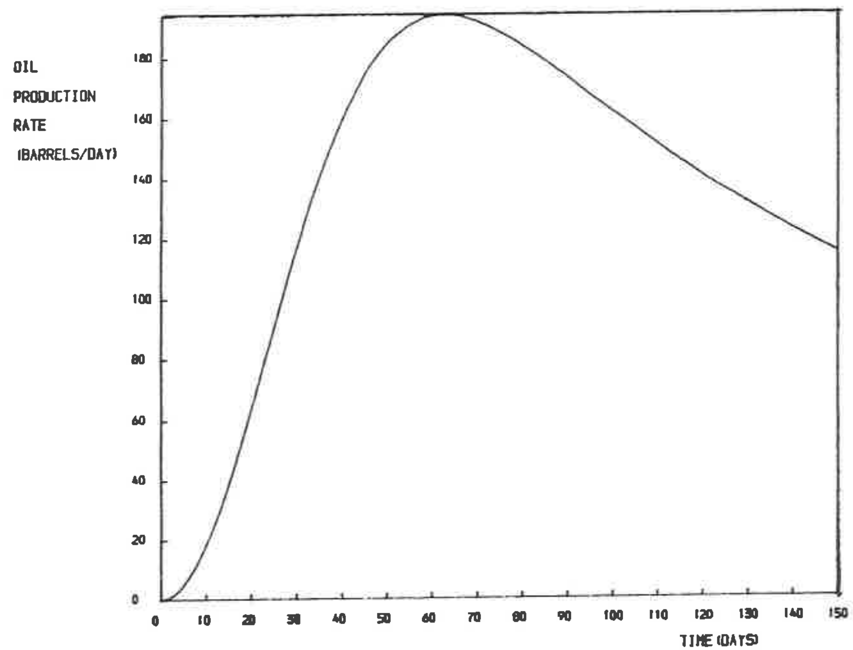
Length of injection period (days).	Rate of steam injection (lbs/day).	Time of maximum rate of oil production (to the nearest day)	Value of maximum oil production rate (barrels/day)
1	500	8	247
2	500	20	220
3	500	33	208
4	500	48	199
5	500	62	194
6	500	78	189
7	500	93	186
8	500	110	183

TABLE 3

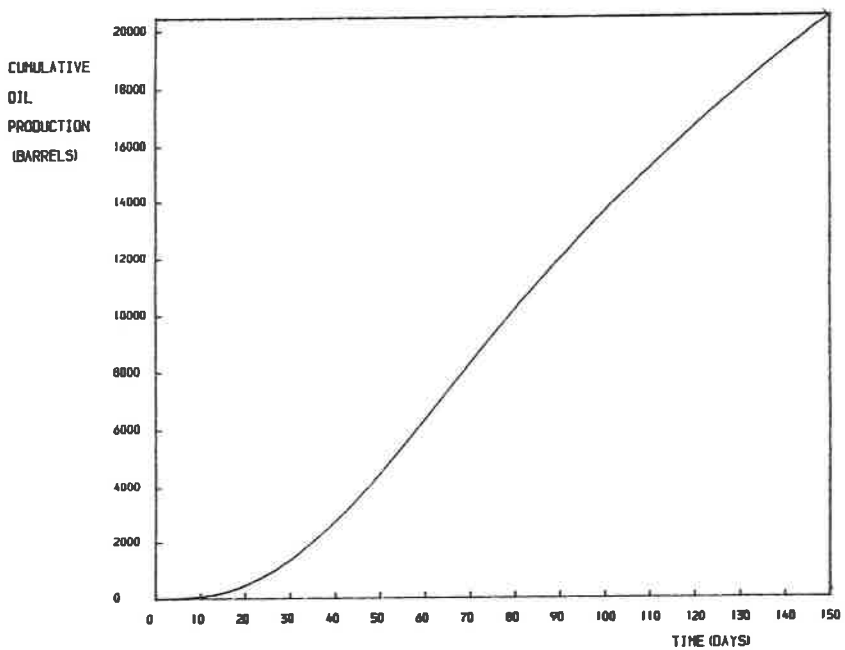
STEAM INJECTION RATE
500 LBS/DAY



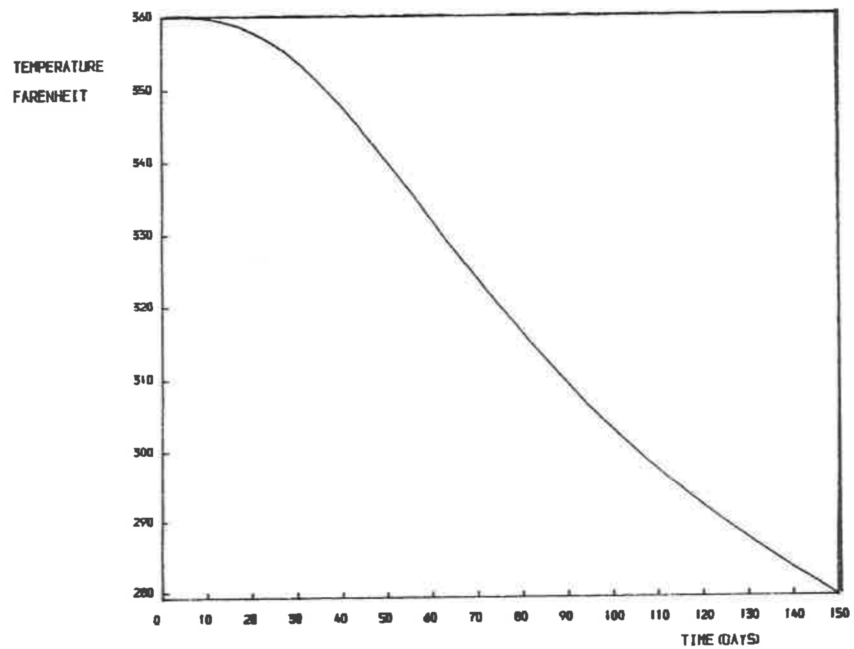
GRAPH 1



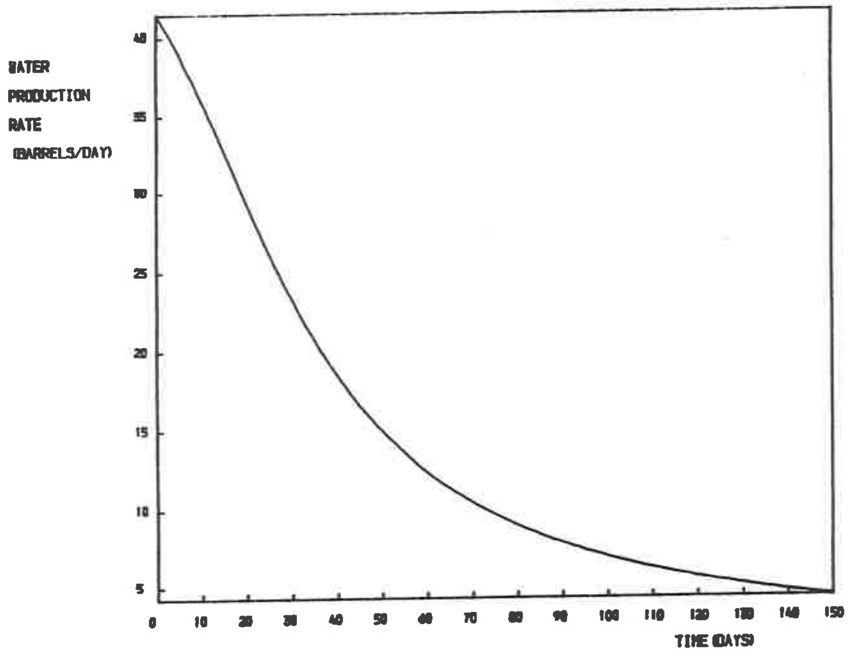
GRAPH 2



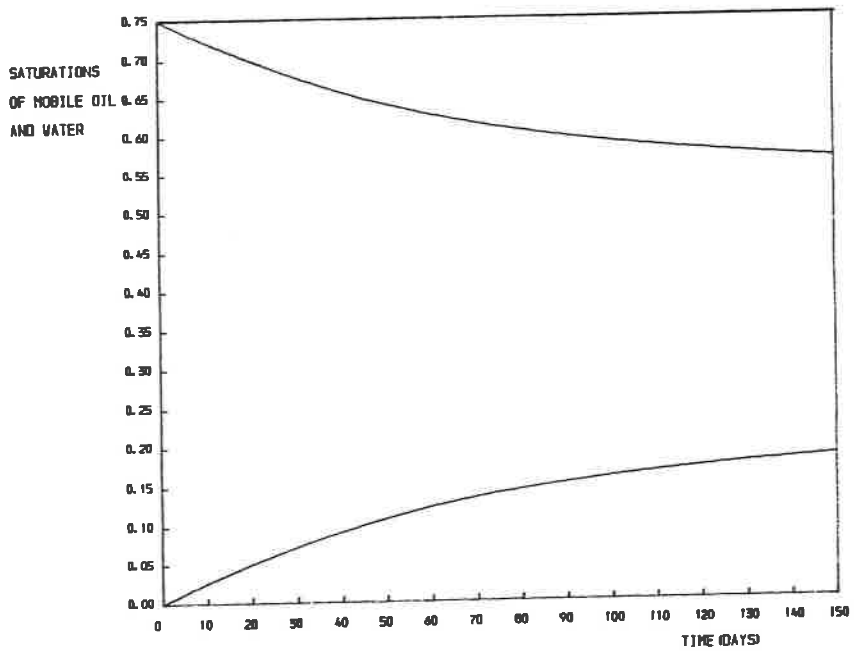
GRAPH 3



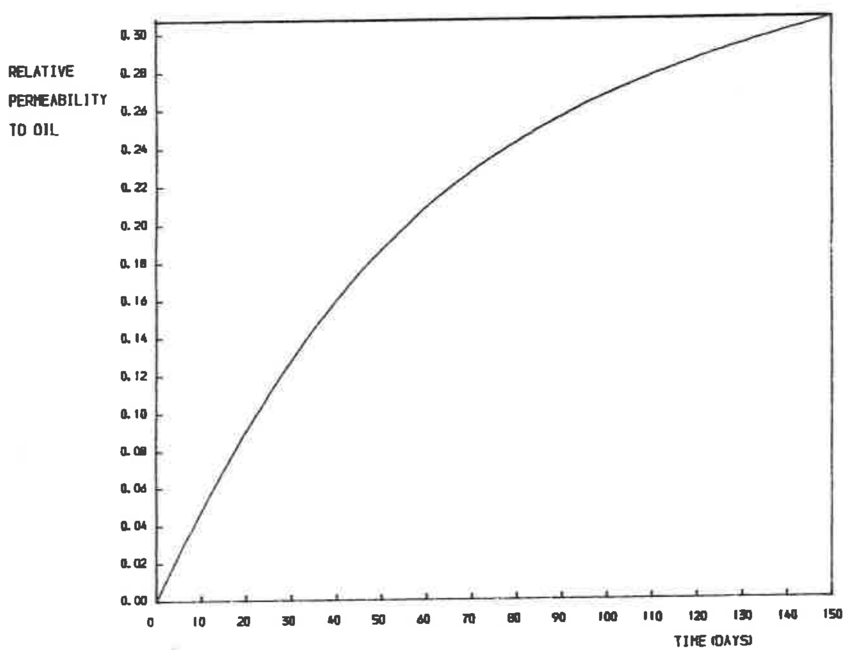
GRAPH 4



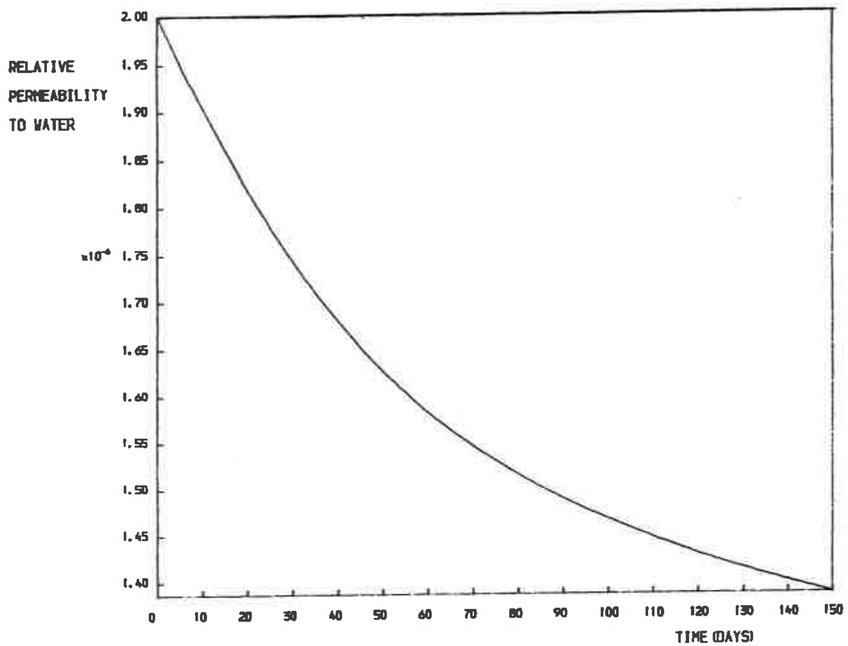
GRAPH 5



GRAPH 6



GRAPH 7



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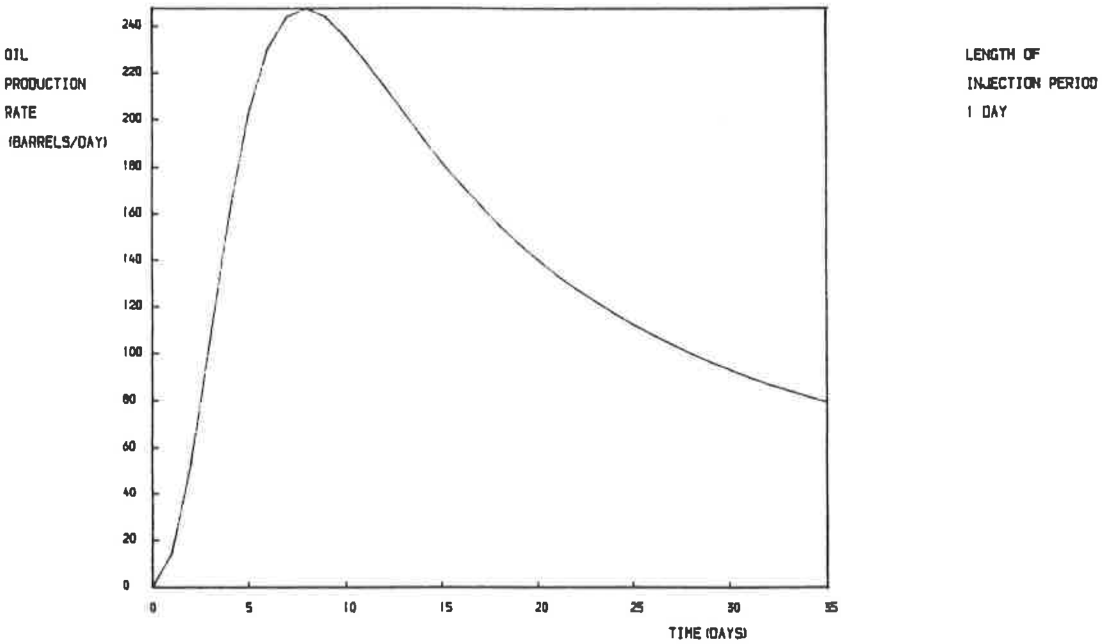

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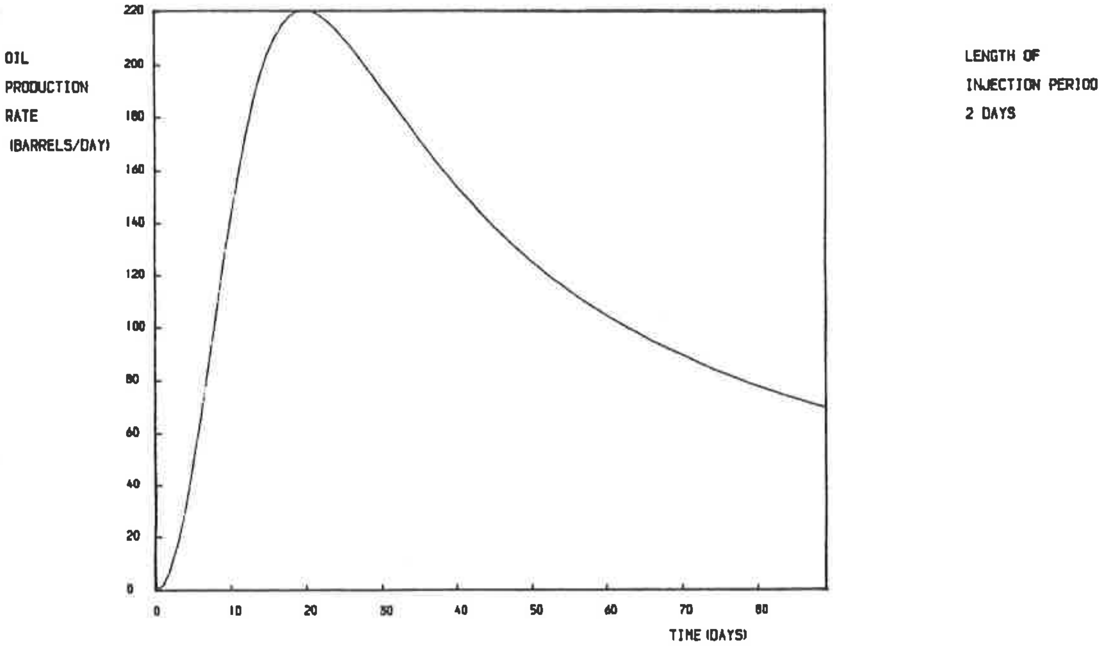
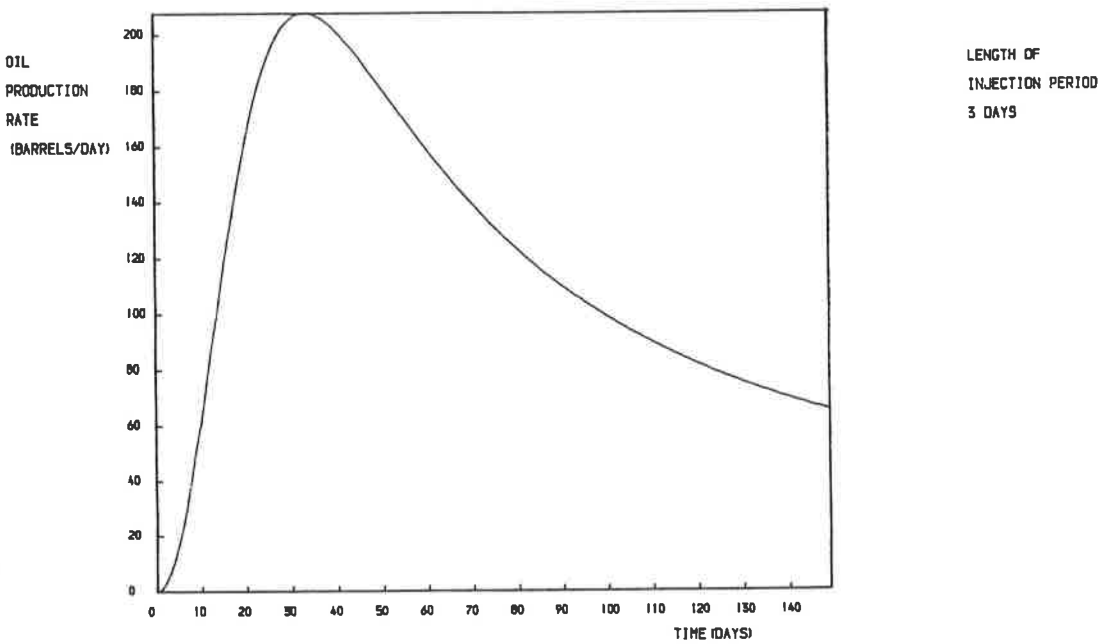
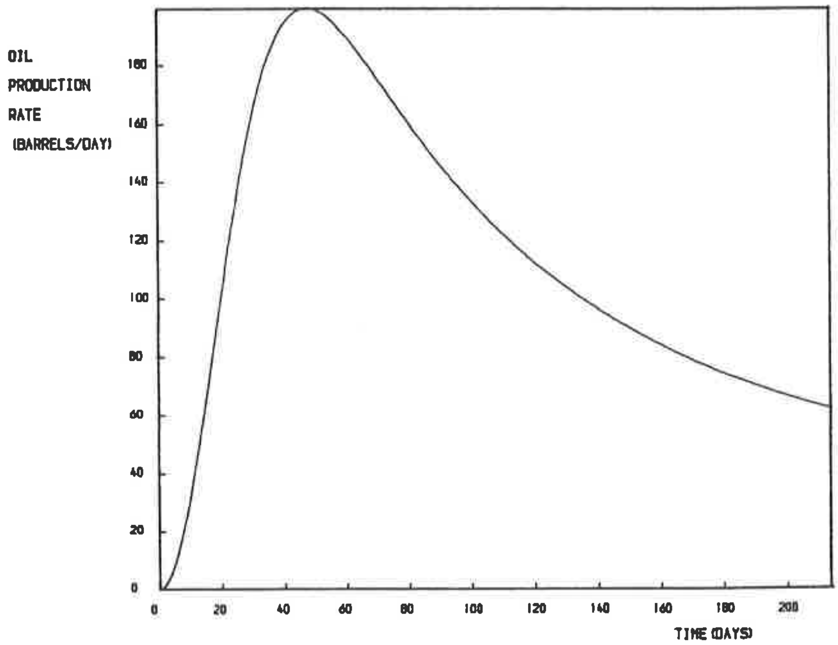


GRAPH 10

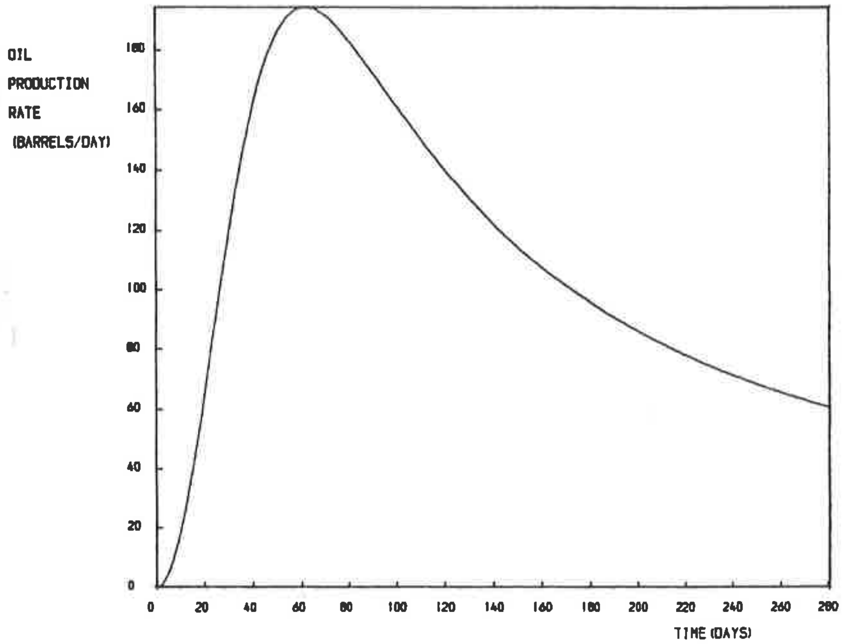


GRAPH 11



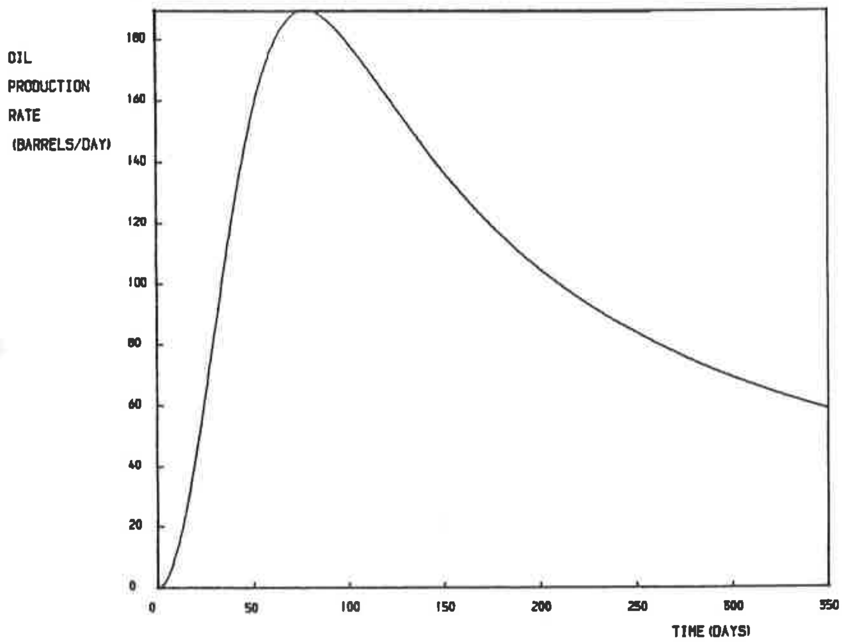
LENGTH OF
INJECTION PERIOD
4 DAYS

GRAPH 12



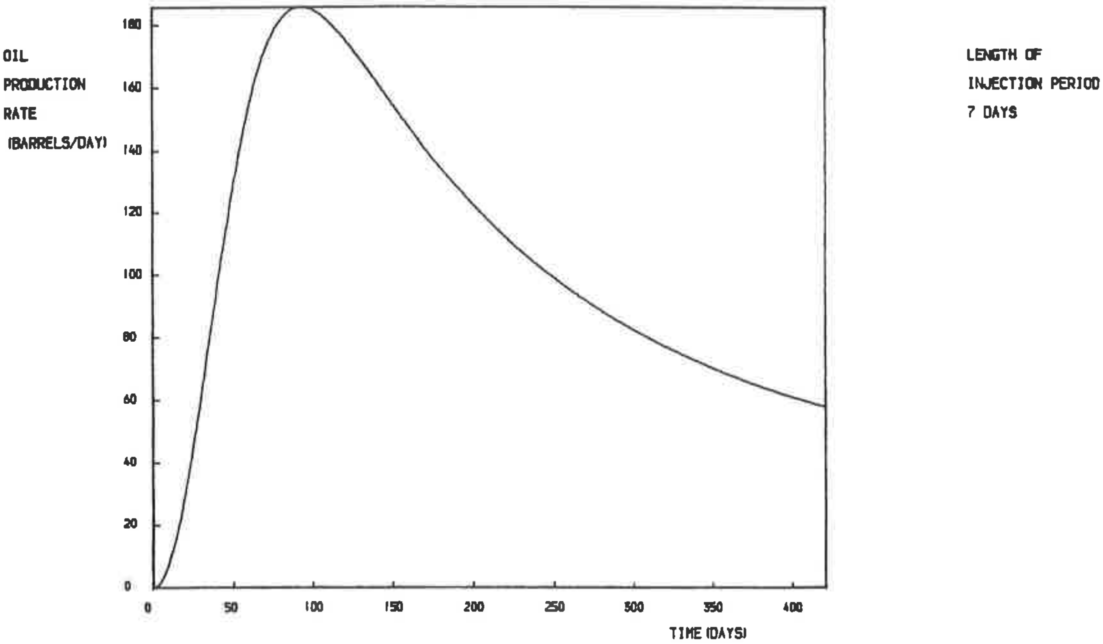
LENGTH OF
INJECTION PERIOD
5 DAYS

GRAPH 13

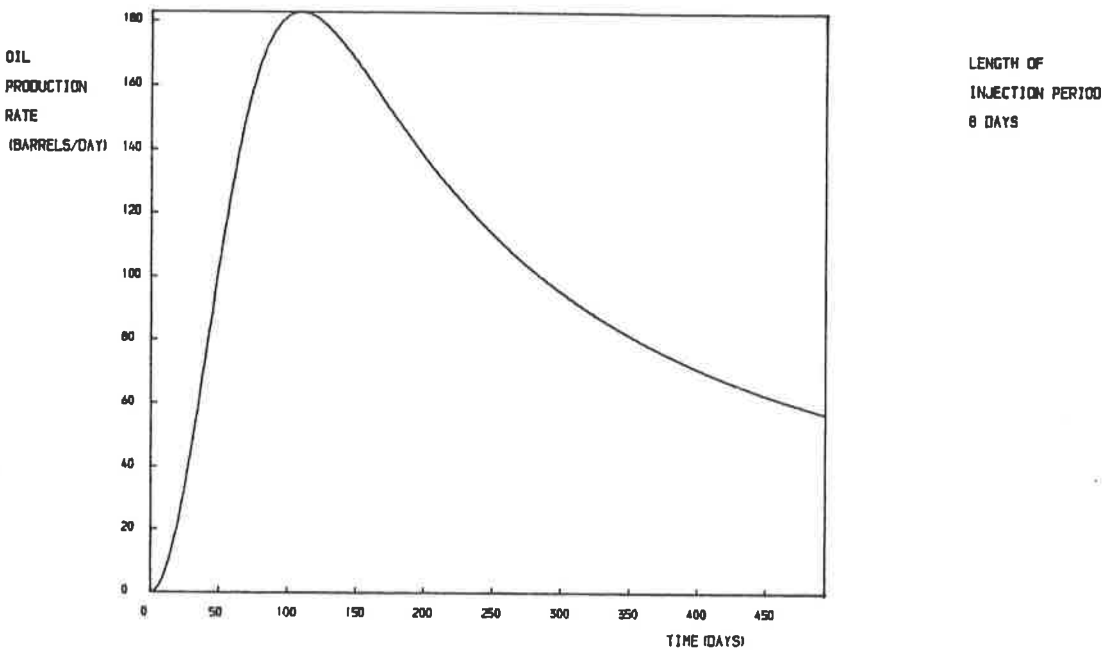


LENGTH OF
INJECTION PERIOD
6 DAYS

GRAPH 14



GRAPH 15



GRAPH 16

D A T A

Reservoir		
	Porosity	ϕ = 0.25
	Horizontal permeability	K_r = 1.5
	Depth	h = 80 ft.
	Radius of well-bore	RW = 0.31 ft.
	Relative permeability to water at $(1 - S_{orw})$	K_{end} = 0.000002
	Density	ρ_r = 167 lb/ft^3
	Specific heat	C_r = 0.21 Btu/lb-°F
	Initial temperature	TR = 110°F
Over/Underlying Rock		
	Thermal conductivity	K_{hf} = 24 Btu/ft-day-°F
	Density	ρ_f = 137 lb/ft^3
	Specific heat	C_f = 0.2 Btu/lb-°F
Steam		
	Latent heat	L_v = 908.8 Btu/lb
	Density	ρ_s = 0.006 lb/ft^3
	Average saturation during injection period	S_{st} = 0.2
	Temperature	TS = 360°F
Water		
	Initial saturation	S_{wi} = 0.2
	Standard temperature	T_{wstd} = 60°F
Oil		
	Residual saturation in the presence of water	S_{orw} = 0.25
	Standard temperature	T_{ostd} = 60°F
	Standard density	ρ_{ostd} = 61.8 lb/ft^3
	Specific heat	C_o = 0.5 Btu/lb
	Viscosity at 100°F	V100 = 5129.68 cs
	Viscosity at 300°F	V300 = 13.26 cs

TABLE 2

7. Summary

A simple mathematical model has been developed to describe the production of heavy oil during the cyclic steam injection process. The variation of the quantities; steam pressure, viscosity and density with temperature has been included in the model along with the change in the saturation of, and relative permeability to, oil and water within the reservoir during the production period.

The numerical algorithm applied to the mathematical model has been described and some results presented. The results indicate that as the length of the injection period increases the maximum rate of oil production decreases and occurs later.

The next step is to form a profit functional into which this production model can be incorporated and to consider the optimisation of such a functional.

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9. Glossary of Symbols

c	compressibility
C_w	Specific heat capacity of water
h	depth of reservoir
H_A	heat contained in the heated zone after one production timestep
H_B	heat contained in the heated zone before one production timestep
H_P	heat taken out of heated zone via oil and water in one production timestep
K	effective permeability
K_{end}	relative permeability to water at $(1 - S_{orw})$
K_o	relative permeability to oil
K_r	horizontal permeability of reservoir
K_w	relative permeability to water
M_o	volumetric heat capacity of oil
M_w	volumetric heat capacity of water
P	pressure
P_s	steam pressure
q	production rate
q_o	production rate of oil
q_w	production rate of water
r	radius
RI	radius of heated zone
RW	radius of wellbore
S	saturation
S_o	saturation of oil
S_{orw}	residual oil saturation in the presence of water
S_w	saturation of water
S_w^*	normalized water saturation

$\overline{S_w}$	saturation of water at beginning of production period
S_{wi}	initial water saturation
t	time
T	temperature
T_R	initial reservoir temperature
T_s	steam temperature
T_{ostd}	standard temperature for oil
T_{wstd}	standard temperature for water
V	volume
W_P	water produced so far in production period
W1P	volume of water in place at beginning of production period
Δp	pressure drop across heated zone
Δt	length of timestep
μ	dynamic viscosity
M_o	dynamic viscosity of oil
μ_w	dynamic viscosity of water
V_o	kinematic viscosity of oil
ρ_o	density of oil
ρ_{ostd}	standard density of oil
ρ_w	density of water
ϕ	porosity

rvp