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Numerical Analysis Report **2/95**

DEPARTMENT OF MATHEMATICS

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ABSTRACT

This report presents a method for constructing open-channel test problems where the analytic solution to the steady Saint-Venant equation is known. Using this method a wide range of such test problems may be created. In particular it is shown how to construct problems with solutions containing a hydraulic jump. Many example test problems are given and two different computational methods are applied to a selection of these. One of the methods is from a commercial package and the other is a novel method for solving the steady flow problem. The commercial package is shown to be more accurate for wholly subcritical flows, however the other method is shown to be superior for other types of flow. Tables of data are given for the example test problems.

CONTENTS

1 INTRODUCTION

The evaluation of numerical methods for computing the steady state flow in an open channel has been hindered because, until recently, there have been no readily available non-trivial test problems with known analytic solution to the steady Saint-Venant equation. Recently MacDonald (1994) described a method for constructing such test problems. The method, in effect, inverts the problem and asks the following question: Given a hypothetical depth profile throughout the channel reach, what must the bed elevation throughout the channel be in order for this profile to satisfy the steady Saint-Venant equation? This report describes this method in detail and extends the previous work to more general channel cross-sections. Many more example test problems are also given.

Two different computational methods are applied to a selection of the examples and the results compared with the analytic solutions. One of these methods is the backwater solver of the FLUCOMP package which is only designed to solve subcritical flows. The second method is a novel method for the solution of steady open channel flow which will be referred to as the Osher method. The actual numerical scheme has been used previously in many other applications. The advantage of this second method is that it can solve for any type of flow, including flows containing hydraulic jumps and critical sections. The existence of analytic solutions for the test problems enables a definite measure of the performance to be obtained for these or any other numerical methods. For this reason the test problems are very suitable for the rigorous validation of commercial packages both unsteady and steady.

So that any modeller can easily apply their own computational method to the example test problems, data for the problems is supplied in tabular form and is also available in electronic form.

2 BACKGROUND AND THEORY

2.1 The Saint-Venant Equations

The Saint-Venant equations model the gradually varied flow of water in an open channel. A derivation of these equations can be found in Cunge, Holly and Verwey (1980). The mass balance and momentum balance equations are respectively

$$T\frac{\partial y}{\partial t}+\frac{\partial Q}{\partial x}=q, \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\beta Q^2}{A}\right)+gA\frac{\partial y}{\partial x}-gA(S_0-S_f)=0, \tag{2}$$

where x is the distance along channel, t is time, $y(x,t)$ is the depth, $Q(x,t)$ is the discharge, $T(x,y)$ is the free surface width, $A(x,y)$ is the wetted area, $S_0(x)$ is the bed slope, $S_f(x,y)$ is the friction slope, q is the lateral inflow, β is the momentum coefficient and g is the acceleration due to gravity. The bed slope, S_0, is given by

$$S_0=-\frac{dz}{dx}, \tag{3}$$

where $z(x)$ is the bed level, the elevation of the channel bed above some horizontal datum. The friction slope, S_f, is given by

$$S_f=\frac{Q|Q|}{K^2}, \tag{4}$$

where $K(x,y)$ is the conveyance. There are many common formulae for the conveyance, any of which may be used by the method described here, but here Manning's equation

$$K=\frac{A^{5/3}}{nP^{2/3}} \tag{5}$$

is used. $P(x,y)$ is the wetted perimeter and n is the Manning friction coefficient.

2.2 Steady Flow

In this work attention is restricted to the steady flow problem, so it is assumed that $y=y(x)$ and $Q=Q(x)$. Under steady flow conditions equations (1) and (2) reduce to the following equations:

$$\frac{dQ}{dx}=q, \tag{6}$$

$$\frac{d}{dx}\left(\frac{\beta Q^2}{A}\right)+gA\frac{dy}{dx}-gA(S_0-S_f)=0. \tag{7}$$

It is assumed that the momentum coefficient (β) and the energy coefficient (α) are both unity. Also the lateral inflow (q) is assumed to be zero. The method described here may be modified so as to be applicable even without these assumptions. For no lateral inflow equation (6) now becomes trivial with solution Q=constant. From now on it is assumed that x is measured in the direction of this constant flow and hence $Q \geq 0$. Now expanding the momentum derivative in equation (7) and dividing by gA yields the equation:

$$\left(1-\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}-\frac{Q^2}{gA^3}\frac{\partial A}{\partial x}-S_0+S_f=0\,. \qquad (8)$$

This is the form of the steady Saint-Venant equation that will be used in this work.

2.3 Construction of Test Problems with Smooth Solutions

To simplify the notation later, equation (8) can be written as:

$$S_0(x)=f_1(x,y(x))\,y'(x)+f_2(x,y(x))\,, \qquad (9)$$

where

$$f_1(x,y)=1-\frac{Q^2T}{gA^3}, \qquad (10)$$

and

$$f_2(x,y)=\frac{Q^2n^2P^{4/3}}{A^{10/3}}-\frac{Q^2}{gA^3}\frac{\partial A}{\partial x}\,. \qquad (11)$$

A test problem with smooth analytical solution in the reach $0 \leq x \leq L$, can be constructed by the following six steps:

1. Specify the functions T, P (and hence A) for $0 \leq x \leq L$. So that differential equation (8) is valid, a certain amount of smoothness is required from the functions T and P. It is more than sufficient to require that T, P, $\partial T/\partial x$ and $\partial T/\partial y$ are all continuous for $0 \leq x \leq L$ and $0 < y \leq y_{\max}$, where $y_{\max}$ is the greatest depth required.

2. Choose a value for the discharge ($Q \geq 0$) and the roughness coefficient (n) for the reach.

3. Choose a hypothetical depth profile, $\hat{y}(x)$, for the reach. This function must satisfy $0<\hat{y}(x) \leq y_{\max}$ and have a continuous first derivative.

4. Define the bed slope of the channel in the reach by

$$S_0(x)=f_1(x,\hat{y}(x))\,\hat{y}'(x)+f_2(x,\hat{y}(x))\,. \qquad (12)$$

5. For many computational models the bed level rather than the bed slope is required. This cannot be found analytically, however, it can be found numerically to a high accuracy. This topic is discussed in Appendix B.

6. If $f_1(0,\hat{y}(0))<0$, i.e. the hypothetical flow is supercritical at inflow, then a depth of $\hat{y}(0)$ must be specified at inflow. If $f_1(L,\hat{y}(L))>0$, i.e. the hypothetical flow is subcritical at outflow, then a depth $\hat{y}(L)$ must be specified at outflow.

It is not hard to see that, by definition, $y \equiv \hat{y}$ satisfies differential equation (9) and hence also equation (8). A complete test problem may be given by the channel defined by T and P, the bed slope given by (12), the discharge, the friction coefficient and the boundary conditions from step 6. The exact solution to differential equation (8) for this problem is $y \equiv \hat{y}$.

2.4 Problems with Solutions Containing a Hydraulic Jump

In order to construct a solution with a hydraulic jump at $x=x^*$, where $0<x^*<L$, write $\hat{y}$ as

$$\hat{y}(x) = \begin{cases} \hat{y}_L(x) & 0 \le x \le x^* \\ \hat{y}_R(x) & x^* < x \le L, \end{cases} \tag{13}$$

where $\hat{y}_L$ and $\hat{y}_R$ are both positive and have continuous first derivatives on the intervals $[0,x^*]$, $[x^*,L]$ respectively. The bed slope can now be calculated as follows:

$$S_0(x) = \begin{cases} S_{0L}(x) & 0 \le x \le x^* \\ S_{0R}(x) & x^* < x \le L, \end{cases} \tag{14}$$

where

$$S_{0L}(x) = f_1(x,\hat{y}_L(x))\,\hat{y}_L'(x) + f_2(x,\hat{y}_L(x)) \tag{15}$$

and

$$S_{0R}(x) = f_1(x,\hat{y}_R(x))\,\hat{y}_R'(x) + f_2(x,\hat{y}_R(x)). \tag{16}$$

The function $\hat{y}$ satisfies differential equation (8) everywhere except at $x=x^*$. To replace the differential equation at this point the jump condition

$$F(x^*,\hat{y}_L(x^*)) = F(x^*,\hat{y}_R(x^*)), \tag{17}$$

is required to be satisfied, where F is the Specific Force given by

$$F(x,y) = \frac{\beta Q^2}{gA} + \int_0^y (y-\eta)\,T(x,\eta)\,d\eta. \tag{18}$$

For a channel of rectangular cross-section, an explicit equation giving the depth on the right of the jump for a particular depth on the left can be found (See Chow (1959)). Most other cross-sections require solving (17) numerically. The jump must also satisfy the thermodynamic law that the total energy cannot increase across the jump. This can be written as

$$E(x^*,\hat{y}_L(x^*)) \ge E(x^*,\hat{y}_R(x^*)), \tag{19}$$

where E is the specific energy given by

$$E(x,y) = \frac{\alpha Q^2}{2gA^2} + y \tag{20}$$

and α is the energy coefficient. For most of the common channel cross-sections, including rectangular and trapezoidal, condition (19) holds if and only if the jump is from supercritical to subcritical. For other cross-sections the condition must be checked for each individual case.

The next thing to consider is the smoothness of the resulting bed slope. In general, using the above construction, the bed slope will be discontinuous at the jump, i.e.

$$S_{0L}(x^*) \neq S_{0R}(x^*)\ . \tag{21}$$

At first thought this may appear perfectly acceptable since many valid test problems have a bed slope discontinuity at some point, however in general this discontinuity does not coincide exactly with the position of the jump. It is desirable to construct test problems where the hydraulic jump does not coincide with a bed slope discontinuity. This can be achieved as follows: Having chosen values for $\hat{y}_L(x^*)$ and $\hat{y}_R(x^*)$ that satisfy both the jump condition and the energy constraint, find values for $\hat{y}_L'(x^*)$ and $\hat{y}_R'(x^*)$ that satisfy the following linear equation:

$$\begin{aligned} S_{0L}(x^*) &= f_1(x^*,\hat{y}_L(x^*))\hat{y}_L'(x^*) + f_2(x^*,\hat{y}_L(x^*)) \\ &= f_1(x^*,\hat{y}_R(x^*))\hat{y}_R'(x^*) + f_2(x^*,\hat{y}_R(x^*)) = S_{0R}(x^*) \end{aligned}\ . \tag{22}$$

For the test problems constructed in this work it has been further required that the bed slope is differentiable at the jump. This requirement might seem unnecessary, however it is required so as to apply analytical results in MacDonald, Baines and Nichols (1994). This report describes a new approach for solving the steady Saint-Venant equations. Under certain assumptions about the bed slope, including the requirement of differentiability, several analytical results are presented, relating to the analytical equations and a given numerical scheme. This scheme is the Osher method which will be applied later. The requirement that the bed slope be differentiable at the jump is met in a similar manner to the previous condition. If $\hat{y}_L(x^*)$, $\hat{y}_R(x^*)$, $\hat{y}_L'(x^*)$ and $\hat{y}_R'(x^*)$ are chosen as described previously. Values for $\hat{y}_L''(x^*)$ and $\hat{y}_R''(x^*)$ are now chosen to satisfy the linear relationship:

$$\begin{aligned} S_{0L}'(x^*) &= f_1(x^*,\hat{y}_L(x^*))\hat{y}_L''(x^*) + \gamma(x^*,\hat{y}_L(x^*).\hat{y}_L'(x^*)) \\ &= f_1(x^*,\hat{y}_R(x^*))\hat{y}_R''(x^*) + \gamma(x^*,\hat{y}_R(x^*).\hat{y}_R'(x^*)) = S_{0R}'(x^*) \end{aligned}\ , \tag{23}$$

where

$$\gamma(x,y,y') = \frac{\partial f_1}{\partial y}(x,y)\,(y')^2 + \left(\frac{\partial f_1}{\partial x}(x,y) + \frac{\partial f_2}{\partial y}(x,y)\right)y' + \frac{\partial f_2}{\partial x}(x,y)\ . \tag{24}$$

Note: In order to apply the above condition, the channel functions must be smooth enough for all the partial derivatives needed in (24) to exist.

3 TEST PROBLEM LIBRARY

In this section the details of a range of test problems produced using the procedures in section 2 are given. The test problems can be divided into different categories depending on the cross-section of the channel and how it changes along the channel reach. By uniform it is meant that the cross-section is constant throughout the channel reach. Conversely by varying it is meant that the cross-section varies along the channel reach. Test problems are included for the following categories:

1. Uniform rectangular (UR1..UR11)

2. Uniform trapezoidal (UT1..UT4)

3. Varying rectangular (VR1..VR4)

4. Uniform exponential (UE1..UE3)

For each category examples are included with purely subcritical or supercritical solutions as well as problems with mixed sub-supercritical solutions. As mentioned in the previous section all the test problems are constructed so that the bed slope is differentiable everywhere along the reach. The resulting bed level is then twice differentiable everywhere. Figures are included for all the test problems. These figures show the stage height (upper solid line), critical stage (dashed line) and the bed level (lower solid line).

3.1 Uniform Rectangular Channel

In this category the channel is described by the functions

$$T(x,y)=B, \tag{25}$$

$$A(x,y)=By, \tag{26}$$

and

$$P(x,y)=B+2y, \tag{27}$$

where B is the given width. The bed slope of the channel is given by the formula

$$S_0(x)=\left(1-\frac{Q^2}{gB^2\hat{y}(x)^3}\right)\hat{y}'(x)+Q^2n^2\frac{(2\hat{y}(x)+B)^{4/3}}{(B\hat{y}(x))^{10/3}}, \tag{28}$$

where Q is the given discharge, n is the given friction coefficient and $\hat{y}$, $\hat{y}'$ are given. For each test problem the appropriate boundary conditions are specified and the corresponding analytic solution is given by $y(x)\equiv\hat{y}(x)$.

UR1: Subcritical

The analytic solution to this test problem is a purely subcritical flow. The stage and bed level are shown in Figure 1. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\left(\frac{4}{g}\right)^{1/3}\left(1+\frac{1}{2}\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right)\right), \tag{29}$$

with

$$\hat{y}'(x)=-\left(\frac{4}{g}\right)^{1/3}\frac{2}{125}\left(\frac{x}{1000}-\frac{1}{2}\right)\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right). \tag{30}$$

The depth at the downstream boundary is 0.748409m.

UR2: Supercritical

The analytic solution to this test problem is a purely supercritical flow. The stage and bed level are shown in Figure 2. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x)=\left(\frac{4}{g}\right)^{1/3}\left(1-\frac{1}{5}\exp\left(-36\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right)\right), \tag{31}$$

with

$$\hat{y}'(x)=\left(\frac{4}{g}\right)^{1/3}\frac{9}{625}\left(\frac{x}{1000}-\frac{1}{2}\right)\exp\left(-36\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right). \tag{32}$$

The depth at the upstream boundary is 0.741599m.

UR3: Subcritical

The analytic solution to this test problem is a oscillatory subcritical flow. The stage and bed level are shown in Figure 3. The test problem is described by the following: L=5000m, B=10m, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\left(\frac{4}{g}\right)^{1/3}\left(\frac{3}{2}+\frac{1}{3}\sin\left(\frac{\pi x}{500}\right)\right), \tag{33}$$

with

$$\hat{y}'(x)=\left(\frac{4}{g}\right)^{1/3}\frac{\pi}{1500}\cos\left(\frac{\pi x}{500}\right). \tag{34}$$

The depth at the downstream boundary is 1.112426m.

UR4: Subcritical

The analytic solution to this test problem is a backwater type flow. The stage and bed level are shown in Figure 4. The test problem is described by the following: L=5000m, B=10m, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\left(\frac{4}{g}\right)^{1/3}\left(\frac{7}{6}+\frac{3}{2}\exp\left(\frac{x-5000}{2500}\right)\right), \tag{35}$$

with

$$\hat{y}'(x)=\left(\frac{4}{g}\right)^{1/3}\frac{3}{5000}\exp\left(\frac{x-5000}{2500}\right). \tag{36}$$

The depth at the downstream boundary is 1.977646m.

UR5: Supercritical

The analytic solution to this test problem is a purely supercritical flow. The stage and bed level are shown in Figure 5. The test problem is described by the following: L=5000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \left(\frac{4}{g}\right)^{1/3}\left(\frac{19}{20} - \frac{1}{6}\exp\left(\frac{-3x}{5000}\right) \right), \tag{37}$$

with

$$\hat{y}'(x) = \left(\frac{4}{g}\right)^{1/3} \frac{1}{10000}\exp\left(\frac{-3x}{5000}\right). \tag{38}$$

The depth at the upstream boundary is 0.580933m.

UR6: Subcritical → Supercritical

The analytic solution to this test problem is subcritical at inflow and smoothly changes to supercritical half way along the reach. The stage and bed level are shown in Figure 6. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(1 - \frac{1}{3}\tanh\left(3\left(\frac{x}{1000} - \frac{1}{2}\right)\right) \right) & 0 \le x \le 500 \\ \left(\frac{4}{g}\right)^{1/3}\left(1 - \frac{1}{6}\tanh\left(6\left(\frac{x}{1000} - \frac{1}{2}\right)\right) \right) & 500 < x \le 1000 \end{cases}, \tag{39}$$

with

$$\hat{y}'(x) = \begin{cases} -\left(\frac{4}{g}\right)^{1/3} \frac{1}{1000}\operatorname{sech}^2\left(3\left(\frac{x}{1000} - \frac{1}{2}\right)\right) & 0 \le x \le 500 \\ -\left(\frac{4}{g}\right)^{1/3} \frac{1}{1000}\operatorname{sech}^2\left(6\left(\frac{x}{1000} - \frac{1}{2}\right)\right) & 500 < x \le 1000 \end{cases}. \tag{40}$$

This problem requires no boundary conditions.

UR7: Supercritical → Subcritical

The analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the reach. The stage and bed level are shown in Figure 7. The test problem is described by the following: L=1000m, B=10m, Q=20m$_3$/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(\frac{9}{10} - \frac{1}{6}\exp\left(\frac{-x}{250}\right) \right) & 0 \le x \le 500 \\ y_2(x) & 500 < x \le 1000 \end{cases}, \tag{41}$$

with

$$\hat{y}'(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3} \frac{1}{1500} \exp\left(\frac{-x}{250}\right) & 0 \le x \le 500 \\ y_2'(x) & 500 < x \le 1000 \end{cases}, \tag{42}$$

where

$$y_2(x) = \left(\frac{4}{g}\right)^{1/3}\left[a_2 + b_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + c_2 \exp\left(-40\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + d_2 \exp\left(-60\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right], \tag{43}$$

and

$$y_2'(x) = -\left(\frac{4}{g}\right)^{1/3} \frac{1}{50}\left[b_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 2c_2 \exp\left(-40\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 3d_2 \exp\left(-60\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right]. \tag{44}$$

Here a_2=2.000000, b_2=-1.831494, c_2=1.998332 and d_2=-1.033357. The depth at the upstream boundary is 0.543853m and the depth at downstream boundary is 1.483173m.

UR8: Supercritical → Subcritical

As in the previous test problem, the analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. The stage and bed level are shown in Figure 8. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(\frac{9}{10} - \frac{1}{6} \exp\left(\frac{-x}{250}\right) \right) & 0 \le x \le 500 \\ y_2(x) & 500 < x \le 1000 \end{cases}, \tag{45}$$

with

$$\hat{y}'(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3} \frac{1}{1500} \exp\left(\frac{-x}{250}\right) & 0 \le x \le 500 \\ y_2'(x) & 500 < x \le 1000 \end{cases}, \tag{46}$$

where

$$y_2(x) = \left(\frac{4}{g}\right)^{1/3}\left[a_2 + b_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + c_2 \exp\left(-40\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + d_2 \exp\left(-60\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right] + \left(\frac{4}{g}\right)^{1/3} e_2 \exp\left(\frac{x}{1000} - 1\right), \tag{47}$$

and

$$y_2'(x) = -\left(\frac{4}{g}\right)^{1/3} \frac{1}{50}\left[b_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 2c_2 \exp\left(-40\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 3d_2 \exp\left(-60\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right] + \left(\frac{4}{g}\right)^{4/3} \frac{e_2}{1000} \exp\left(\frac{x}{1000} - 1\right). \tag{48}$$

Here a_2=1.000000, b_2=-0.348427, c_2=0.552264, d_2=-0.555580 and

e_2=0.800000. The depth at the upstream boundary is 0.543853m and the depth at downstream boundary is 1.334899m.

UR9: Subcritical → Supercritical → Subcritical.

The analytic solution to this test problem is subcritical at inflow and changes smoothly to supercritical at x=300m and back to subcritical, via a hydraulic jump at x=600m. The stage and bed level are shown in Figure 9. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(1-\tanh\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 0 \le x \le 300 \\ \left(\frac{4}{g}\right)^{1/3}\left(1-\frac{1}{6}\tanh\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right)\right) & 300 < x \le 600 \\ y_3(x) & 600 < x \le 1000 \end{cases}, \tag{49}$$

with

$$\hat{y}'(x) = \begin{cases} -\left(\frac{4}{g}\right)^{1/3}\frac{1}{1000}\operatorname{sech}^2\left(\frac{x}{1000}-\frac{3}{10}\right) & 0 \le x \le 300 \\ -\left(\frac{4}{g}\right)^{1/3}\frac{1}{1000}\operatorname{sech}^2\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 300 < x \le 600 \\ y_3'(x) & 600 < x \le 1000 \end{cases}, \tag{50}$$

where

$$y_3(x) = \left(\frac{4}{g}\right)^{1/3}\left[a_3 + b_3\exp\left(-20\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + c_3\exp\left(-40\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + d_3\exp\left(-60\left(\frac{x}{1000}-\frac{3}{5}\right)\right)\right] + \left(\frac{4}{g}\right)^{1/3} e_3\exp\left(\frac{x}{1000}-1\right), \tag{51}$$

and

$$y_3'(x) = -\left(\frac{4}{g}\right)^{1/3}\frac{1}{50}\left[b_3\exp\left(-20\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + 2c_3\exp\left(-40\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + 3d_3\exp\left(-60\left(\frac{x}{1000}-\frac{3}{5}\right)\right)\right] + \left(\frac{4}{g}\right)^{1/3}\frac{e_3}{1000}\exp\left(\frac{x}{1000}-1\right). \tag{52}$$

Here a_3=1.000000, b_3=0.033139, c_3=-0.138978, d_3=-0.254000 and e_3=0.800000. The depth at the downstream boundary is 1.334919m.

UR10: Subcritical → Supercritical → Subcritical → Supercritical

The analytic solution to this test problem is subcritical at inflow and changes smoothly to supercritical at x=300m and back to subcritical, via a hydraulic jump at x=600m. The flow then changes smoothly to supercritical again at about 800m. The stage and bed level are shown in Figure 10. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(1-\tanh\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 0 \le x \le 300 \\ \left(\frac{4}{g}\right)^{1/3}\left(1-\frac{1}{6}\tanh\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right)\right) & 300 < x \le 600 \\ y_3(x) & 600 < x \le 1000 \end{cases}, \quad (53)$$

with

$$\hat{y}'(x) = \begin{cases} -\left(\frac{4}{g}\right)^{1/3}\frac{1}{1000}\mathrm{sech}^2\left(\frac{x}{1000}-\frac{3}{10}\right) & 0 \le x \le 300 \\ -\left(\frac{4}{g}\right)^{1/3}\frac{1}{1000}\mathrm{sech}^2\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 300 < x \le 600 \\ y_3'(x) & 600 < x \le 1000 \end{cases}, \quad (54)$$

where

$$y_3(x) = \left(\frac{4}{g}\right)^{1/3}\left[a_3 + b_3\exp\left(-10\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + c_3\exp\left(-20\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + d_3\exp\left(-30\left(\frac{x}{1000}-\frac{3}{5}\right)\right)\right], \quad (55)$$

and

$$y_3'(x) = -\left(\frac{4}{g}\right)^{1/3}\frac{1}{100}\left[b_3\exp\left(-10\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + 2c_3\exp\left(-20\left(\frac{x}{1000}-\frac{3}{5}\right)\right) + 3d_3\exp\left(-30\left(\frac{x}{1000}-\frac{3}{5}\right)\right)\right]. \quad (56)$$

Here a_3=0.900000, b_3=0.382537, c_3=2.131437 and d_3=-2.237556. This test problem requires no boundary conditions.

UR11: Supercritical → Subcritical → Supercritical

The analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. It then soon changes smoothly back to supercritical. The stage and bed level are shown in Figure 11. The test problem is described by the following: L=1000m, B=10m, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3}\left(\frac{9}{10}-\frac{1}{6}\exp\left(\frac{-x}{250}\right)\right) & 0 \le x \le 500 \\ y_2(x) & 500 < x \le 1000 \end{cases}, \quad (57)$$

with

$$\hat{y}'(x) = \begin{cases} \left(\frac{4}{g}\right)^{1/3} \frac{1}{1500} \exp\left(\frac{-x}{250}\right) & 0 \le x \le 500 \\ y_2'(x) & 500 < x \le 1000 \end{cases}, \tag{58}$$

where

$$y_2(x) = \left(\frac{4}{g}\right)^{1/3} \left[a_2 + b_2 \exp\left(-10\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + c_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + d_2 \exp\left(-30\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right], \tag{59}$$

and

$$y_2'(x) = -\left(\frac{4}{g}\right)^{1/3} \frac{1}{100} \left[b_2 \exp\left(-10\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 2c_2 \exp\left(-20\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 3d_2 \exp\left(-30\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right]. \tag{60}$$

Here a_2=0.900000, b_2=-0.901807, c_2=4.373858 and d_2=-3.238569. The depth at the upstream boundary is 0.543853m.

3.2 Uniform Trapezoidal Channel

In this category the channel is described by the functions

$$T(x,y)=B+2yz, \tag{61}$$

$$A(x,y)=y(B+yz), \tag{62}$$

and

$$P(x,y)=B+2y\sqrt{1+z^2}, \tag{63}$$

where B is the bottom width and z is the side slope of the channel. The bed slope of the channel is given by the formula

$$S_0(x)=\left(1-\frac{Q^2(B+2\hat{y}(x)z)}{g(B+\hat{y}(x)z)^3\hat{y}(x)^3}\right)\hat{y}'(x)+\frac{Q^2n^2\left(B+2\hat{y}(x)\sqrt{1+z^2}\right)^{4/3}}{(B+\hat{y}(x)z)^{10/3}\hat{y}(x)^{10/3}}, \tag{64}$$

where Q is the given discharge, n is the given friction coefficient and $\hat{y}$, $\hat{y}'$ are given. For each test problem the appropriate boundary conditions are specified and the corresponding analytic solution is given by $y(x)\equiv\hat{y}(x)$.

UT1: Subcritical

The analytic solution to this test problem is a purely subcritical flow. The stage and bed level are shown in Figure 12. The test problem is described by the following: L=1000m, B=9m, z=2, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=0.751+0.3755\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right), \tag{65}$$

with

$$\hat{y}'(x)=-0.012016\left(\frac{x}{1000}-\frac{1}{2}\right)\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right). \tag{66}$$

The depth at the downstream boundary is 0.757878m.

UT2: Subcritical

The analytic solution to this test problem is a oscillatory subcritical flow. The stage and bed level are shown in Figure 13. The test problem is described by the following: L=5000m, B=10m, z=2, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\frac{9}{8}+\frac{1}{4}\sin\left(\frac{\pi x}{500}\right), \tag{67}$$

and

$$\hat{y}'(x)=\frac{\pi}{2000}\cos\left(\frac{\pi x}{500}\right). \tag{68}$$

The depth at the downstream boundary is 1.125m.

UT3: Supercritical → Subcritical

The analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. The stage and bed level are shown in Figure 14. The test problem is described by the following: L=1000m, B=10m, z=1, Q=20m^3/s, n=0.02,

$$\hat{y}(x)=\begin{cases}\frac{27}{40}-\frac{1}{8}\exp\left(\frac{-x}{250}\right) & 0\le x\le 500\\ y_2(x) & 500<x\le 1000\end{cases}, \tag{69}$$

with

$$\hat{y}'(x)=\begin{cases}\frac{1}{2000}\exp\left(\frac{-x}{250}\right) & 0\le x\le 500\\ y_2'(x) & 500<x\le 1000\end{cases}, \tag{70}$$

where

$$\begin{aligned}y_2(x)=a_2+b_2\exp\left(-30\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)+c_2\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\\ +d_2\exp\left(-90\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)+e_2\exp\left(\tfrac{x}{1000}-1\right),\end{aligned} \tag{71}$$

and

$$\begin{aligned}y_2'(x)=-\tfrac{3}{100}\Big[b_2\exp\left(-30\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)+2c_2\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\\ +3d_2\exp\left(-90\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\Big]+\tfrac{e_2}{1000}\exp\left(\tfrac{x}{1000}-1\right).\end{aligned} \tag{72}$$

Here a_2=0.750000, b_2=-0.562186, c_2=0.556864, d_2=-0.315774 and e_2=0.600000. The depth at the upstream boundary is 0.55m and the depth at downstream boundary is 1.35m.

UT4: Subcritical → Supercritical → Subcritical.

The analytic solution to this test problem is subcritical at inflow and changes smoothly to supercritical at x=300m and back to subcritical, via a hydraulic jump, at x=600m. The stage and bed level are shown in Figure 15. The test problem is described by the following: L=1000m, B=10m, z=1, Q=20m^3/s, n=0.02,

$$\hat{y}(x)=\begin{cases}0.723449\left(1-\tanh\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 0\le x\le 300\\ 0.723449\left(1-\frac{1}{6}\tanh\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right)\right) & 300<x\le 600\\ y_3(x) & 600<x\le 1000\end{cases}, \tag{73}$$

with

$$\hat{y}'(x)=\begin{cases} -0.000723449\,\mathrm{sech}^2\left(\frac{x}{1000}-\frac{3}{10}\right) & 0\le x\le 300 \\ -0.000723449\,\mathrm{sech}^2\left(6\left(\frac{x}{1000}-\frac{3}{10}\right)\right) & 300<x\le 600\ , \\ y_3'(x) & 600<x\le 1000 \end{cases} \tag{74}$$

where

$$\begin{aligned} y_3(x)=a_3+b_3\exp\left(-20\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right)+c_3\exp\left(-40\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right) \\ +d_3\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right)+e_3\exp\left(\tfrac{x}{1000}-1\right), \end{aligned} \tag{75}$$

and

$$\begin{aligned} y_3'(x)=\tfrac{-1}{50}\Big[b_3\exp\left(-20\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right)+2c_3\exp\left(-40\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right) \\ +3d_3\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{3}{5}\right)\right)\Big]+\tfrac{e_2}{1000}\exp\left(\tfrac{x}{1000}-1\right). \end{aligned} \tag{76}$$

Here a_3=0.750000, b_3-0.111051, c_3=0.026876, d_3=-0.217567 and e_3=0.600000. The depth the downstream boundary is 1.349963m.

3.3 Varying Rectangular Channel

In this category the channel is described by the functions

$$T(x,y)=B(x), \tag{77}$$

$$A(x,y)=B(x)y, \tag{78}$$

and

$$P(x,y)=B(x)+2y, \tag{79}$$

where $B(x)$ is the given width at distance x along the reach. The bed slope of the channel is given by the formula

$$S_0(x)=\left(1-\frac{Q^2}{gB(x)^2\hat{y}(x)^3}\right)\hat{y}'(x)+\frac{Q^2n^2(2\hat{y}(x)+B(x))^{4/3}}{(B(x)\hat{y}(x))^{10/3}}-\frac{Q^2B'(x)}{gB(x)^3\hat{y}(x)^2}, \tag{80}$$

where Q is the given discharge, n is the given friction coefficient and $\hat{y}$, $\hat{y}'$ are given. For each test problem the appropriate boundary conditions are specified and the corresponding analytic solution is given by $y(x)\equiv\hat{y}(x)$.

VR1: Subcritical.

The analytic solution to this test problem is a purely subcritical flow. The width profile for the channel is shown in Figure 16 and the stage and bed level are shown in Figure 17. The test problem is described by the following: L=5000m, Q=20m^3/s, n=0.03,

$$B(x)=10-5\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)^2\right)-5\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)^2\right),$$

with

$$B'(x)=\tfrac{16}{125}\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)^2\right)+\tfrac{16}{125}\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)^2\right),$$

and

$$\hat{y}(x)=1+\tfrac{1}{2}\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)^2\right)+\tfrac{1}{2}\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)^2\right),$$

with

$$\hat{y}'(x)=-\tfrac{8}{625}\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{1}{3}\right)^2\right)-\tfrac{8}{625}\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)\exp\left(-64\left(\tfrac{x}{5000}-\tfrac{2}{3}\right)^2\right),$$

The depth at the downstream boundary is 1.000408m.

VR2: Subcritical.

The analytic solution to this test problem is a purely subcritical flow. The width profile of the channel is shown in Figure 18 and the stage and bed level are shown in Figure 19. The test problem is described by the following: L=1000m, Q=20m^3/s,

n=0.03,

$$B(x)=10-80\left(\left(\tfrac{x}{1000}\right)^2-2\left(\tfrac{x}{1000}\right)^3+\left(\tfrac{x}{1000}\right)^4\right),$$

with

$$B'(x)=-\tfrac{4}{25}\left(\left(\tfrac{x}{1000}\right)-3\left(\tfrac{x}{1000}\right)^2+2\left(\tfrac{x}{1000}\right)^3\right),$$

and

$$\hat{y}(x)=1+8\left(\left(\tfrac{x}{1000}\right)^2-2\left(\tfrac{x}{1000}\right)^3+\left(\tfrac{x}{1000}\right)^4\right),$$

with

$$\hat{y}'(x)=\tfrac{2}{125}\left(\left(\tfrac{x}{1000}\right)-3\left(\tfrac{x}{1000}\right)^2+2\left(\tfrac{x}{1000}\right)^3\right).$$

The depth at the downstream boundary is 1m.

VR3: Supercritical $\rightarrow$ Subcritical.

The analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. The width profile of the channel is shown in Figure 20 and the stage and bed level are shown in Figure 21. The test problem is described by the following: L=1000m, Q=20m^3/s, n=0.02,

$$B(x)=10-64\left(\left(\tfrac{x}{1000}\right)^2-2\left(\tfrac{x}{1000}\right)^3+\left(\tfrac{x}{1000}\right)^4\right), \quad (89)$$

with

$$B'(x)=-\tfrac{16}{125}\left(\left(\tfrac{x}{1000}\right)-3\left(\tfrac{x}{1000}\right)^2+2\left(\tfrac{x}{1000}\right)^3\right), \quad (90)$$

and

$$\hat{y}(x)=\begin{cases} -\dfrac{1}{40}+\dfrac{1}{1+2\left(\frac{x}{1000}-\frac{1}{2}\right)^2} & 0\le x\le 500 \\ y_2(x) & 500<x\le 1000 \end{cases}, \quad (91)$$

with

$$\hat{y}'(x)=\begin{cases} \dfrac{\frac{-1}{250}\left(\frac{x}{1000}-\frac{1}{2}\right)}{\left(1+2\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right)^2} & 0\le x\le 500 \\ y_2'(x) & 500<x\le 1000 \end{cases}, \quad (92)$$

where

$$y_2(x)=a_2+b_2\exp\left(-30\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)+c_2\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)$$
$$+d_2\exp\left(-90\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\ +\ e_2\exp\left(\tfrac{x}{4000}-\tfrac{1}{4}\right), \tag{93}$$

and

$$y_2'(x)=-\tfrac{3}{100}\left[b_2\exp\left(-30\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)+2c_2\exp\left(-60\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\right.$$
$$\left.+3d_2\exp\left(-90\left(\tfrac{x}{1000}-\tfrac{1}{2}\right)\right)\right]\ +\ \tfrac{e_2}{4000}\exp\left(\tfrac{x}{4000}-\tfrac{1}{4}\right), \tag{94}$$

Here a_2=0.000000, b_2=0.769035, c_2=-0.755596, d_2=0.106813 and e_2=1.125000. The depth at the upstream boundary is 0.641667m and the depth at downstream boundary is 1.125m.

VR4: Supercritical → Subcritical.

Like the previous example, the analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. The width profile for the channel is shown in Figure 20 and the stage and bed level are shown in Figure 22. The test problem is described by the following: L=1000m, Q=20m^3/s, n=0.02,

$$B(x)=10-64\left(\left(\tfrac{x}{1000}\right)^2-2\left(\tfrac{x}{1000}\right)^3+\left(\tfrac{x}{1000}\right)^4\right), \tag{95}$$

with

$$B'(x)=-\tfrac{16}{125}\left(\left(\tfrac{x}{1000}\right)-3\left(\tfrac{x}{1000}\right)^2+2\left(\tfrac{x}{1000}\right)^3\right), \tag{96}$$

and

$$\hat{y}(x)=\begin{cases}-\dfrac{1}{40}+\dfrac{1}{1+2\left(\frac{x}{1000}-\frac{1}{2}\right)^2} & 0\le x\le 500\\[2ex] y_2(x) & 500< x\le 1000\end{cases}, \tag{97}$$

with

$$\hat{y}'(x)=\begin{cases}\dfrac{\frac{-1}{250}\left(\frac{x}{1000}-\frac{1}{2}\right)}{\left(1+2\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right)^2} & 0\le x\le 500\\[2ex] y_2'(x) & 500< x\le 1000\end{cases}, \tag{98}$$

where

$$y_2(x) = a_2 + b_2\exp\left(-30\left(\frac{x}{1000}-\frac{1}{2}\right)\right) + c_2\exp\left(-60\left(\frac{x}{1000}-\frac{1}{2}\right)\right) + d_2\exp\left(-90\left(\frac{x}{1000}-\frac{1}{2}\right)\right) + e_2\exp\left(\frac{x}{4000}-\frac{1}{4}\right), \tag{99}$$

and

$$y_2'(x) = -\frac{3}{100}\left[b_2\exp\left(-30\left(\frac{x}{1000}-\frac{1}{2}\right)\right) + 2c_2\exp\left(-60\left(\frac{x}{1000}-\frac{1}{2}\right)\right) + 3d_2\exp\left(-90\left(\frac{x}{1000}-\frac{1}{2}\right)\right)\right] + \frac{e_2}{4000}\exp\left(\frac{x}{4000}-\frac{1}{4}\right), \tag{100}$$

Here a_2=0.000000, b_2=-0.230680, c_2=0.248267, d_2=-0.228271 and e_2=1.500000. The depth at the upstream boundary is 0.641667m and the depth at the downstream boundary is 1.5m.

3.4 Uniform Exponential Channel

In this category the channel is described by the functions

$$T(x,y)=B\exp(\lambda y), \tag{101}$$

$$A(x,y)=\frac{B}{\lambda}(\exp(\lambda y)-1), \tag{102}$$

and

$$\begin{aligned} P(x,y)=B-&\frac{\sqrt{4+B^2\lambda^2}}{\lambda}+\frac{\sqrt{4+B^2\lambda^2\exp(2\lambda y)}}{\lambda} \\ &+\frac{2}{\lambda}\ln\left(\frac{2+\sqrt{4+B^2\lambda^2}}{B\lambda}\right)-\frac{2}{\lambda}\ln\left(\frac{2+\sqrt{4+B^2\lambda^2\exp(2\lambda y)}}{B\lambda\exp(\lambda y)}\right), \end{aligned} \tag{103}$$

where B is the bottom width and λ is a parameter that controls the rate at which the channel width increases with depth. The bed slope of the channel is given by the formula

$$S_0(x)=\left(1-\frac{Q^2T(x,\hat{y}(x))}{gA(x,\hat{y}(x))^3}\right)\hat{y}'(x)+\frac{Q^2n^2P(x,\hat{y}(x))^{4/3}}{A(x,\hat{y}(x))^{10/3}}, \tag{104}$$

where Q is the given discharge, n is the given friction coefficient and $\hat{y}$, $\hat{y}'$ are given. For each test problem the appropriate boundary conditions are specified and the corresponding analytic solution is given by $y(x)\equiv\hat{y}(x)$.

UE1: Subcritical.

The analytic solution to this test problem is a purely subcritical flow. The channel cross-section is shown in Figure 23 and the stage and bed level are shown in Figure 24. The test problem is described by the following: L=1000m, B=5m, λ=2, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\frac{9}{10}+\frac{9}{20}\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right), \tag{105}$$

with

$$\hat{y}'(x)=-\frac{9}{625}\left(\frac{x}{1000}-\frac{1}{2}\right)\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right). \tag{106}$$

The depth at the downstream boundary is 0.908242m.

UE2: Subcritical.

Like the previous problem, the analytic solution to this test problem is a purely subcritical flow, however, in this case the flow is significantly deeper and so the surface width is much greater. The channel cross-section is shown in Figure 23 and the stage and bed level are shown in Figure 25 The test problem is described by the following: L=1000m, B=5m, λ=2, Q=20m^3/s, n=0.03,

$$\hat{y}(x)=\frac{3}{2}+\frac{3}{4}\exp\left(-16\left(\frac{x}{1000}-\frac{1}{2}\right)^2\right), \tag{107}$$

with

$$\hat{y}'(x) = -\frac{3}{125}\left(\frac{x}{1000} - \frac{1}{2}\right)\exp\left(-16\left(\frac{x}{1000} - \frac{1}{2}\right)^2\right). \tag{108}$$

The depth at the downstream boundary is 1.513737m.

UE3: Supercritical → Subcritical.

The analytic solution to this test problem is supercritical at inflow and changes, via a hydraulic jump, to subcritical half way along the channel reach. The channel cross-section is shown in Figure 23 and the stage and bed level are shown in Figure 26. The test problem is described by the following: L=1000m, B=5m, λ=2, Q=20m^3/s, n=0.02,

$$\hat{y}(x) = \begin{cases} \frac{3}{4} - \frac{1}{10}\exp\left(\frac{-x}{250}\right) & 0 \le x \le 500 \\ y_2(x) & 500 < x \le 1000 \end{cases}, \tag{109}$$

with

$$\hat{y}'(x) = \begin{cases} \frac{1}{2500}\exp\left(\frac{-x}{250}\right) & 0 \le x \le 500 \\ y_2'(x) & 500 < x \le 1000 \end{cases}, \tag{110}$$

where

$$\begin{aligned} y_2(x) = {} & a_2 + b_2\exp\left(-35\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + c_2\exp\left(-70\left(\frac{x}{1000} - \frac{1}{2}\right)\right) \\ & + d_2\exp\left(-105\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + e_2\exp\left(\frac{3x}{4000} - \frac{3}{4}\right), \end{aligned} \tag{111}$$

and

$$\begin{aligned} y_2'(x) = {} & -\frac{7}{200}\left[b_2\exp\left(-35\left(\frac{x}{1000} - \frac{1}{2}\right)\right) + 2c_2\exp\left(-70\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right. \\ & \left. + 3d_2\exp\left(-105\left(\frac{x}{1000} - \frac{1}{2}\right)\right)\right] + \frac{3e_2}{4000}\exp\left(\frac{3x}{4000} - \frac{3}{4}\right). \end{aligned} \tag{112}$$

Here a_2=0.750000, b_2=-0.705439, c_2=0.645958, d_2=-0.323826 and e_2=0.750000. The depth at the upstream boundary is 0.65m and the depth at the downstream boundary is 1.5m.

4 COMPUTATIONAL EXPERIMENTS

In this section two different computational methods are applied to a selection of the test problems given in section 3. A brief description of the two methods is first given.

4.1 FLUCOMP

FLUCOMP is a package developed by HR Wallingford Ltd for the one-dimensional modelling of natural rivers. Full details of this package can be found in Samuels and Gray (1982). The package solves unsteady flow, however there is a separate module for the computation of steady flow. This is only designed to compute backwater profiles and hence cannot cope with supercritical flow. The backwater solver uses the trapezium rule applied to the dynamic equation in form (2) to solve from one section to the next in the upstream direction and hence the method is second order accurate. For the test problems in this work, the channels are described by continuous functions. However, because the package is designed for natural rivers, the channel must be described by discrete measurements. This adds an extra level of approximation to the solution process. The cross-section of the channel is supplied at discrete points along the reach. Each of these cross-sections is made up of a number of discrete offset/ ground level measurements. The solution is calculated at each point along the reach where section data is supplied. The choice of position of each cross-section and the number of measurements per section will effect the performance of the method. For each section, where data is given, the package constructs a table of values for certain functions such as the wetted area, momentum coefficient, energy coefficient and friction function at regular intervals of depth up to a given maximum depth. This table is then used to compute values for these functions at any arbitrary depth using linear interpolation. The number of points in the table, for a given maximum depth, will also effect the performance of the method. In this work the maximum number of points, 40, was used.

In section 2 it was assumed that the momentum and energy coefficients are unity. FLUCOMP automatically tries to compute these coefficients so there can be no control over them. It was observed that for the rectangular channels FLUCOMP gives both these coefficients values of unity and for the other cross-sections they were very near to unity. The results from FLUCOMP should not differ significantly from the results obtained if these coefficients were set to exactly one.

4.2 Osher

This is a new method for the solution of steady flow and is described in MacDonald, Baines and Nichols (1994). The numerical scheme discretises the steady Saint-Venant equation in conservation form using the well known Enguist Osher approximation to the derivative (see Engquist and Osher (1981)).

Because the equation is discretised in conservation form any discontinuities will be correct. This is a similar situation to the theory of numerical schemes for first order conservation laws such as Inviscid Burgers equation. Unlike FLUCOMP, which starts at the downstream boundary and proceeds in the upstream directions, Osher solves for the entire channel reach simultaneously and unlike FLUCOMP can be supplied with an upstream boundary condition. Also, unlike FLUCOMP, Osher works with the continuous functions that describe the channel and hence no approximation to the channel is necessary. The method can solve for any type of flow including multiple transitions. In MacDonald, Baines and Nichols (1994) it was demonstrated that the method was very effective at solving flows with hydraulic jumps. At present the method is only first order accurate so it is expected that the FLUCOMP method would be more accurate in the case of purely subcritical flows. However, the fact that FLUCOMP only solves for an approximation to the channel, in the cases where the cross-section is not rectangular, might reduce its accuracy. Osher can only be applied to prismatic channels at present, although a modification for non-prismatic channels is currently under development. Also under development is a second order accurate extension. It is also hoped that the method can be applied successfully to natural rivers.

4.3 Results

For each type of cross-section a selection of test problems were chosen considering mainly those where the flow is subcritical at the downstream boundary. This was so FLUCOMP could attempt to solve them. For each set of results the numerical solution minus the analytic solution, in mm, is plotted. Hence a positive error indicates that the numerical solution is above the analytic solution and conversely a negative error indicates that it is below the numerical solution. When computing this error both sets of figures were already rounded to the nearest mm and hence there is an uncertainty in the error of plus or minus one mm which is shown by the error bars. For each test problem the same uniform grid of spacing *dx* was used for both Osher and FLUCOMP.

UR1: Subcritical

For both methods a spacing of *dx*=10m was used. Figure 27 shows the errors for FLUCOMP and Figure 28 shows the errors for Osher. The errors from FLUCOMP are very small (less than 2mm) away from the ends of the channel reach, however within 200m of the ends the solution is very poor. This can be easily explained. The critical flow number describes the state of the flow; it is less than one for subcritical flow, greater than one for supercritical flow and exactly one at critical flow. FLUCOMP limits the solution to having a critical flow number of 0.8 or less. If at some point the flow is detected to violate this condition the depth is set to a certain constant value set by the package. It can seen by Figure 1 that the depth approaches very close to critical at both ends of the reach and hence causing FLUCOMP to object. The solution recovers in the middle when the analytic solution no longer violates this critical flow number limit.

Osher, on the other hand, has no problems with this test problem. The errors are slightly greater than FLUCOMP in the centre of the reach (4mm maximum).

UR3: Subcritical

For both methods a spacing of *dx*=25m was used. Figure 29 shows the errors for FLUCOMP and Figure 30 shows the errors for Osher. Both methods have no trouble with this test problem, however FLUCOMP is particularly good (2mm maximum). The structure of the errors for Osher is not surprising since the analytic solution in (Figure 3) is oscillatory. The errors oscillate like the exact solution and errors of greatest magnitude (about -16mm) coincide with the points where the analytic solution is at a peak, hence Osher is undershooting the peaks. For most engineering applications an error of 16mm is still quite acceptable.

UR5: Supercritical

For both methods a spacing of *dx*=25m was used. Figure 31 shows the errors for FLUCOMP and Figure 32 shows the errors for Osher. The analytic solution to this test problem is a supercritical flow (Figure 5). FLUCOMP was given a supercritical value at the downstream boundary. This immediately violates the critical number condition and the rest of the flow is an incorrect subcritical flow. It can be seen in Figure 31 and in other FLUCOMP results that in regions where the method fails, there are one or two spurious errors that break the trend; for example the error of about 50mm in Figure 31. It is suggested that these spurious errors are due to iterations that have failed to converge in the solution procedure of the trapezium rule.
For the Osher method the upstream boundary condition required by the test problem could be supplied and the method has succeeded with a maximum error of 2mm, which is very good.

UR9: Subcritical → Supercritical → Subcritical

For both methods a spacing of *dx*=10m was used. Figure 33 shows the errors for FLUCOMP and Figure 34 shows the errors for Osher. The analytical solution to this problem (Figure 9) is subcritical at the downstream boundary. FLUCOMP follows this solution very well until the solution tries to become supercritical by a hydraulic jump half way along the channel. FLUCOMP then continues with an incorrect subcritical flow. The analytical solution eventually becomes subcritical again and FLUCOMP is able to recover with the errors becoming very small again. Osher gives a correct solution for the whole channel except for a few grid points in the neighbourhood of the hydraulic jump. The large errors near the jump are not serious at all and may be explained by smearing and slightly incorrect jump positioning. It should also be noticed that these errors are only local and do not effect the rest of the solution.

UT2: Subcritical

For both methods a spacing of *dx*=25m was used. Figure 35 shows the errors for FLUCOMP and Figure 36 shows the errors for Osher. This test problem is very similar to UR3 except that the channel is now trapezoidal in cross-section. All the comments for UR3 hold here.

UT4: Subcritical → Supercritical → Subcritical

For both methods a spacing of *dx*=10m was used. Figure 37 shows the errors for FLUCOMP and Figure 38 shows the errors for Osher. This test problem is very similar to UR9 except that the channel is now trapezoidal in cross-section. All the comments for UR9 hold again.

VR1: Subcritical

A spacing of *dx*=25m was used. Figure 39 shows the errors for FLUCOMP. Osher cannot be applied to this test problem at present, because the channel is non-prismatic. The analytic solution is a purely subcritical flow (Figure 17) and FLUCOMP is very successful. The errors are very small with a maximum of 4mm.

VR2: Subcritical

A spacing of *dx*=10m was used. Figure 40 shows the errors for FLUCOMP. Again because the channel is non-prismatic, Osher cannot be applied to this test problem. The analytic solution is again a purely subcritical flow (Figure 19) and FLUCOMP has no problems at all. The errors are very small with a maximum of 4mm.

VR3: Supercritical → Subcritical

A spacing of *dx*=10m was used. Figure 41 shows the errors for FLUCOMP. Again because the channel is non-prismatic, Osher cannot be applied to this test problem at present. The analytic solution to this problem (Figure 21) is subcritical at the downstream boundary and jumps to supercritical half way along the reach. FLUCOMP follows the subcritical profile very well, but after where the jump should be it continues with an incorrect subcritical flow.

UE1: Subcritical

For both methods a spacing of *dx*=10m was used. Figure 42 shows the errors for FLUCOMP and Figure 43 shows the errors for Osher. The analytic solution is again a purely subcritical flow (Figure 24) and both methods succeed. The maximum error for FLUCOMP (4mm) is better than the maximum error for Osher (8mm)

UE2: Subcritical

For both methods a spacing of *dx*=10m was used. Figure 44 shows the errors for FLUCOMP and Figure 45 shows the errors for Osher. The analytic solution is again a purely subcritical flow (Figure 25) and both methods succeed. The maximum error for FLUCOMP (3mm) is significantly better than the maximum error for Osher (21mm)

UE3:Supercritical → Subcritical

For both methods a spacing of *dx*=10m was used. Figure 46 shows the errors for FLUCOMP and Figure 47 shows the errors for Osher. The analytic solution to this problem (Figure 26) is subcritical at the downstream boundary and jumps to supercritical half way along the reach. FLUCOMP follows the subcritical profile very well, but after where the jump should be it continues with an incorrect subcritical flow. On the other hand Osher manages to solve for the flow correctly along the entire reach. As for previous problems with jumps there are large errors only in the locality of the hydraulic jump.

5. CONCLUSIONS

A method has been described for constructing test problems for steady open-channel flow for which the analytic solution to the steady Saint-Venant equation is known. A wide range of such test problems, including many with a hydraulic jump, have been given. For a selection of these test problems two numerical methods have been applied, the FLUCOMP package and the Osher method. The FLUCOMP package can only integrate completely subcritical flow. It has been observed that for these flows the method is very accurate with a maximum observed error of 4mm. However, the method has difficulty if the flow becomes supercritical or subcritical flow becomes too close to critical.

Unlike FLUCOMP, Osher can solve for all types of flow not just the subcritical cases. Away from the immediate location of any hydraulic jumps the accuracy is good, however, not as good as FLUCOMP. The maximum observed error is about 25mm, however this is more than acceptable for the majority of engineering needs. The large errors observed near the hydraulic jumps are due to three reasons: i) the slightly incorrect position of the jump, ii) the smearing of the hydraulic jump and iii) the sharp variation in depth near the jump. These errors are not at all serious since they are only local. At present Osher can only solve for prismatic channels, however a modification for channels of varying cross-section is under development.

The difference in accuracy between FLUCOMP and Osher could be explained by the fact that FLUCOMP uses the trapezium rule and hence is second order accurate, whereas Osher is only first order accurate. A second order modification to Osher is also being developed.

As a final note it is pointed out that it might not be too difficult to modify FLUCOMP so that it could solve for all types of flow while still maintaining the trapezium rule for integration. This could be done by integrating both upstream and downstream from each control and fitting jumps in between the controls, if necessary. Such an algorithm is described in Humpidge and Moss (1971). Another alternative is to implement a version of Osher for natural rivers.

ACKNOWLEDGEMENTS

This work was supported by the EPSRC and HR Wallingford Ltd. Much of this work was carried out at HR Wallingford, who are thanked for supplying the facilities.

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Appendix A: Notation

x	Distance along channel (m).
t	Time (s).
y	Depth (m).
Q	Discharge (m^3/s).
A	Wetted area (m^2).
T	Channel width (m).
P	Wetted perimeter (m).
g	Acceleration due to gravity ($9.80665 m/s^2$).
L	Length of channel reach (m).
z	Level of bed above some horizontal datum (m).
S_0	Bed slope $=-dz/dx$.
S_f	Friction slope.
q	Lateral inflow (m^2/s).
K	Conveyance (m^3/s).
n	Manning friction coefficient.
F	Specific Force (m^3).
E	Specific Energy (m).
α	Energy coefficient.
β	Momentum coefficient.
$\hat{y}$	Hypothetical depth function (m).
h', h''	First and second derivatives of a function h, respectively.
f_1, f_2	Bed slope functions: $S_0=f_1\hat{y}'+f_2$.
x^*	Position of hydraulic jump (m).
$\hat{y}_L$, $\hat{y}_R$	Hypothetical depth functions for left and right of jump (m).
S_{0L}, S_{0R}	Bed slope functions for left and right of jump.
y_{max}	Maximum depth for hypothetical depth functions (m).
B	Width for rectangular channels (m).
η	Dummy integration variable.

Appendix B: Integration of Bed Slope

It was mentioned in section 2 that it is not possible to find an analytic expression for the bed levels of the test problems constructed using the method described. In this Appendix possible methods for numerically computing the bed levels are discussed.

The bed levels and the bed slope are related by the following equation:

$$z'(x) = -S_0(x) \quad 0 \le x \le L, \qquad z(L) = 0, \tag{113}$$

where z is the bed level, S_0 is the bed slope and L is the length of the channel reach. Equation (113) can also be written as

$$z(x) = \int_x^L S_0(t)\,dt, \quad 0 \le x \le L. \tag{114}$$

There are many standard methods for numerically solving form (113), for example see Lambert (1991). Also there are possible methods for solving the equation in form (114). The chosen method should ideally have the following properties: (1) If bed level is computed at discrete points along the channel reach then accuracy of each of these values should be guaranteed to some predetermined tolerance. (2) Using the discrete data computed with the numerical method it should be possible to recover the bed level at any intermediate point to some guaranteed accuracy. Having accurate bed levels is important because all differences between the numerical results and the exact solution should be clearly attributable to the approximate nature of the flow solver and not to errors in bed levels. This is all the more important, because the test problems given in this work are being considered for use as standard benchmark for assessing the performance of particular models. The second property would be very useful so that the computational modeller could be supplied with a data file allowing the accurate computation of bed levels at arbitrary points along the reach. Thus allowing refinement of the computational grid without losing the accuracy of the bed levels.

Integration of Cubic Spline

A method is now given that satisfies both the requirements given above

Given the set of points

$$0 = x_0 < x_1 < \ldots < x_{N-1} < x_N = L, \tag{115}$$

there is a unique function, p, of the form

$$p(x) = d_i x^3 + c_i x^2 + b_i x + a_i \quad x_{i-1} \le x < x_i \qquad i = 1, \ldots, N, \tag{116}$$

that has a continuous second derivative on the interval $0 \le x \le L$, and satisfies the following:

$$p(x_i) = S_0(x_i) \quad i = 0, 1, \ldots, N, \qquad p'(0) = S_0'(0), \quad p'(L) = S_0'(L). \tag{117}$$

The function p is called the simple cubic spline of S_0, and if S_0 has a continuous fourth derivative, it can be shown to satisfy the error bound (See Atkinson (1989)):

$$|p(x) - S_0(x)| \le \frac{5}{384} h_{\max}^4 \max_{0 \le x \le L} |S_0^{(iv)}(x)| \qquad 0 \le x \le L, \tag{118}$$

where

$$h_{\max} = \max_{i=1,..N} (x_i - x_{i-1}) .$$

The method used here is as follows: Substitute the spline of the bed slope into equation (114) instead of the actual bed slope, to obtain:

$$Z(x) = \int_x^L p(t)\,dt, \tag{120}$$

which can be calculated analytically, and is a quartic spline. It is now easy to show the error bound:

$$|Z(x) - z(x)| \le \frac{5}{384} L h_{\max}^4 \max_{0 \le x \le L} |S_0^{(iv)}(x)| \qquad 0 \le x \le L, \tag{121}$$

It appears that by choosing $h_{\max}$ sufficiently small then the readily computable function Z can give an accurate bed level at all points along the channel. In many cases this method works well, however there are some difficulties. The fourth derivative of the bed slope may not be continuous everywhere. In particular it will most likely be discontinuous at any jump. Now to obtain a valid error bound the channel must be split into separate intervals and a separate cubic spline computed for each interval. The error bound (121) depends on the maximum magnitude of the fourth derivative of the bed slope and the largest spacing of interpolation points. If the bed slope varies sharply in some small region, such as near a jump, a large amount of interpolation points may be required to achieve a required accuracy, even though the slope behaves well throughout most of the reach. Again this can be overcome by splitting the reach. It is not easy to calculate a bound for the fourth derivative of the bed slope since the number of terms can grow exponentially as the bed slope is repeatedly differentiated. It is time consuming even when using the symbolic manipulation package Mathematica (Wolfram

(1988)) running on a powerful PC.

Appendix C: Test Problem Data

For test problems UR1, UR3, UR5, UR9, UT2, UT4, VR1, VR2, VR3, UE1, UE2 and UE3 given in section 3, the cubic spline method described in Appendix B was used to compute the bed levels. For UR1, UR3, UR5, UR9, UT2 and UT4, bounds for the fourth derivative of the bed slope were computed and hence a rigorous error bound obtained. The number of interpolation points were chosen so that the bed levels are accurate to 1mm. For the other test problems it was considered too time consuming to compute bounds for the fourth derivative and hence these were estimated from similar examples where bounds are known. For each test problem the coefficients of the piecewise quartic polynomial are stored in an ascii data file. These data files available on a floppy disk. Each file has the format:

N	err_bound				
x_1	a_1	b_1	c_1	d_1	e_1
x_2	a_2	b_2	c_2	d_2	e_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_i	a_i	b_i	c_i	d_i	e_i
x_{i+1}	a_{i+1}	b_{i+1}	c_{i+1}	d_{i+1}	e_{i+1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_N	a_N	b_N	c_N	d_N	e_N

where for $x_{i-1} \le x \le x_i$

$$Z(x) = a_i + b_i\left(\frac{x}{L}\right) + c_i\left(\frac{x}{L}\right)^2 + d_i\left(\frac{x}{L}\right)^3 + e_i\left(\frac{x}{L}\right)^4,$$

for $i=1,...,N$. Here $x_0=0$. If the value of *err_bound* is non zero it gives the bound for the error in the bed level. The Fortran format of the first line is "I8,E14.2" and the rest of the lines have the format "F8.0,5F14.5". Table 1 shows this data file for UR5 and this is the shortest file with only 50 lines of coefficients. Other test problems have data files with up to 1000 lines of coefficients. The data files have the ending "_bl.coef", so for example table 1 is stored in "ur5_bl.coef".

The above data was used to create bed level and stage data tables. These are given by tables 2-13 and are accurate to the nearest mm. For the remaining test problems given in section 3 (UR2, UR4, UR6, UR7, UR8, UR10, UR11, UT1, UT3 and VR4) there was not time to apply the cubic spline method. However, since the test problems were already programmed in the symbolic manipulation package Mathematica (Wolfram (1988)), Mathematica's built in numerical ODE solver was used to create bed level and stage data tables. These are in tables 14-23. A very small error tolerance was used so they should be accurate to the nearest mm, however the reader must find their own method of calculating the bed level at other points. For most purposes it seems that linear interpolation is sufficient.

The bed level and stage level data is available in files ending in "_bl.dat", for example for test problem UR1 the file "ur1_bl.dat".

FIGURES

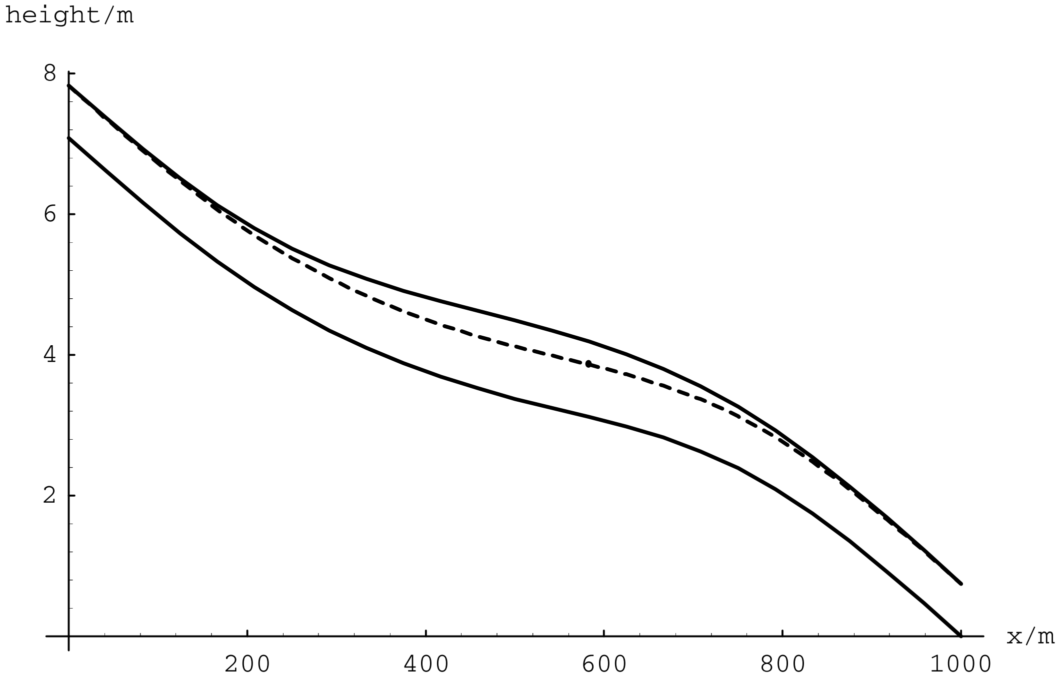

Figure 1: Stage and Bed Level for UR1

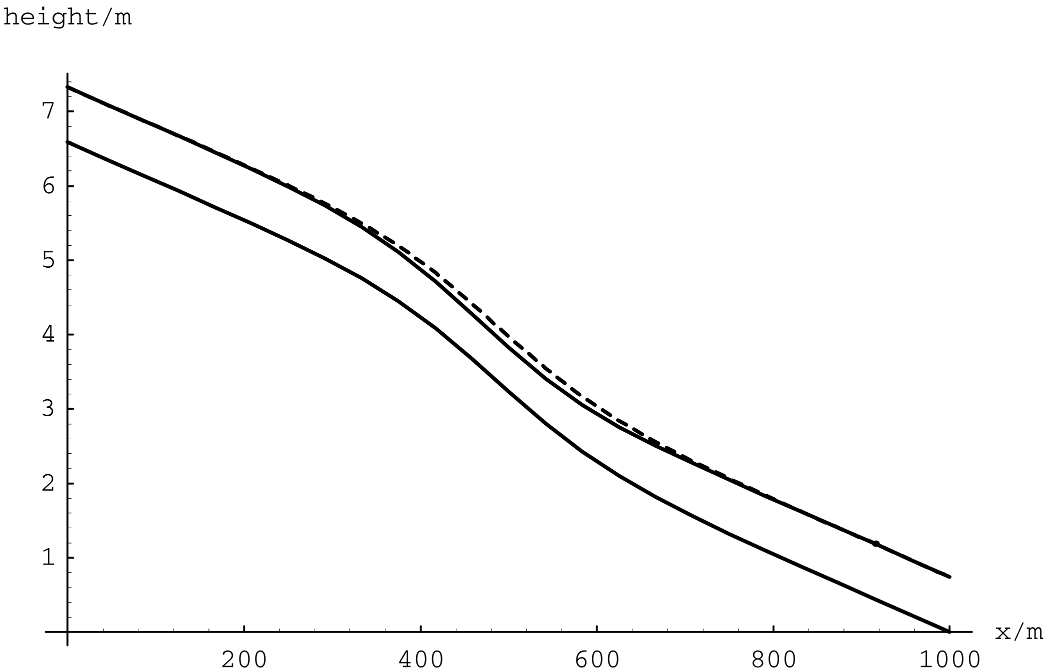

Figure 2: Stage and Bed Level for UR2

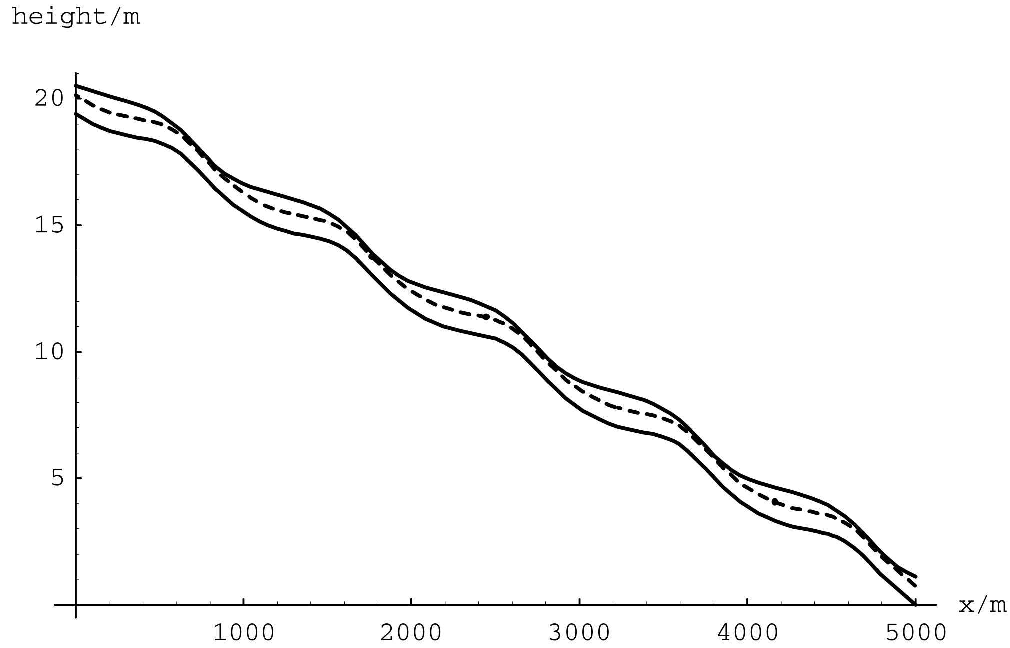

Figure 3: Stage and Bed Level for UR3

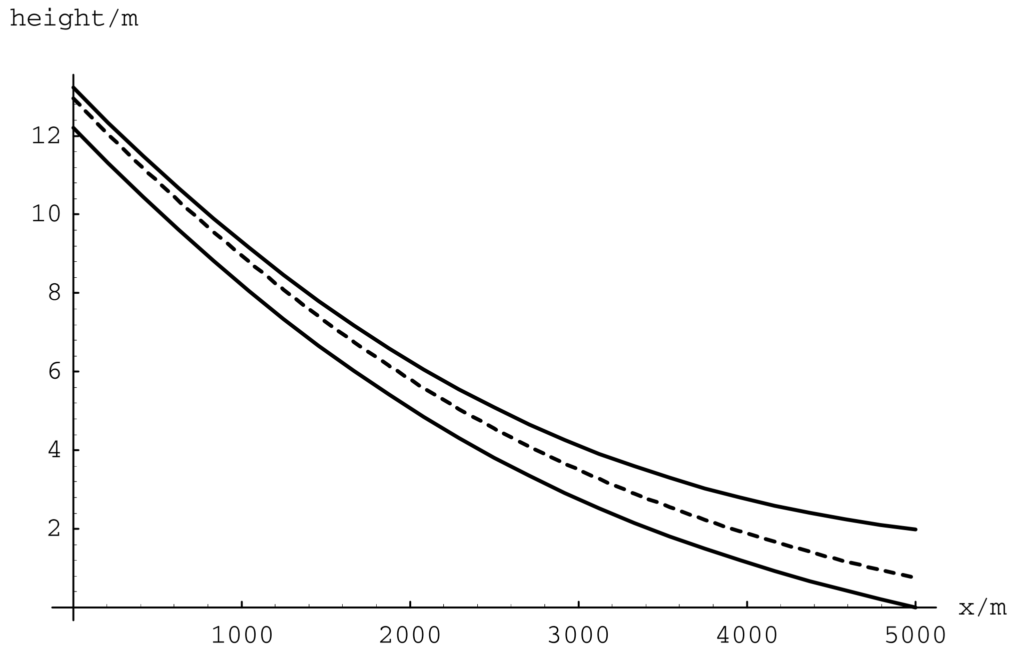

Figure 4: Stage and Bed Level for UR4

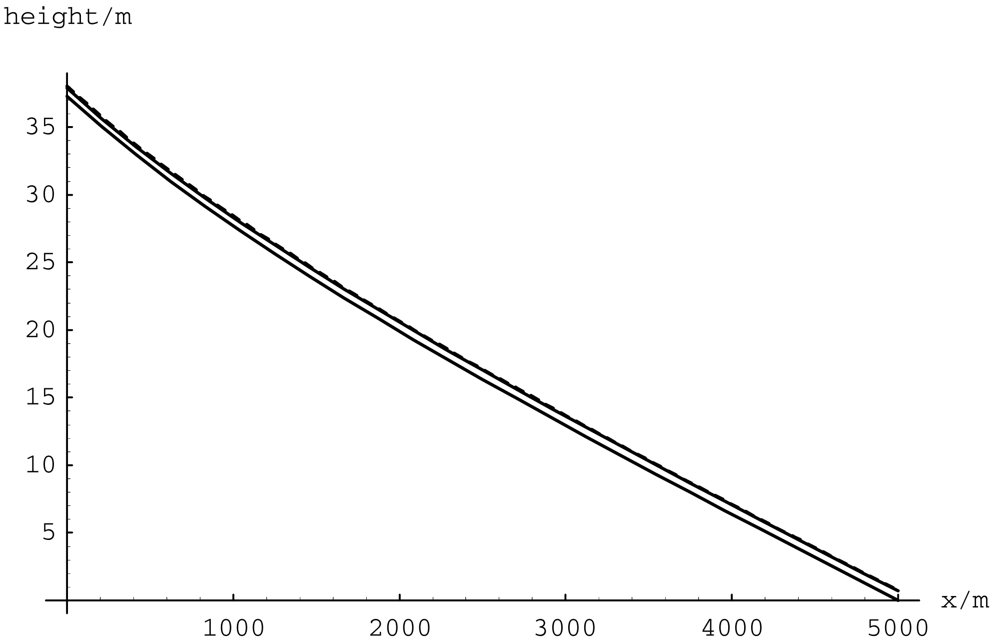

Figure 5: Stage and Bed Level for UR5

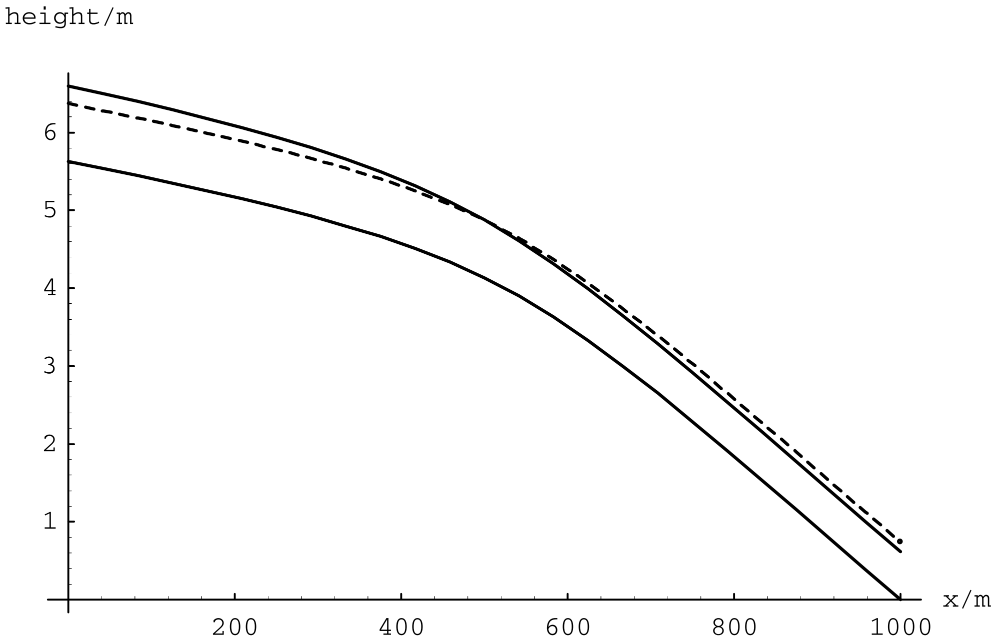

Figure 6: Stage and Bed Level for UR6

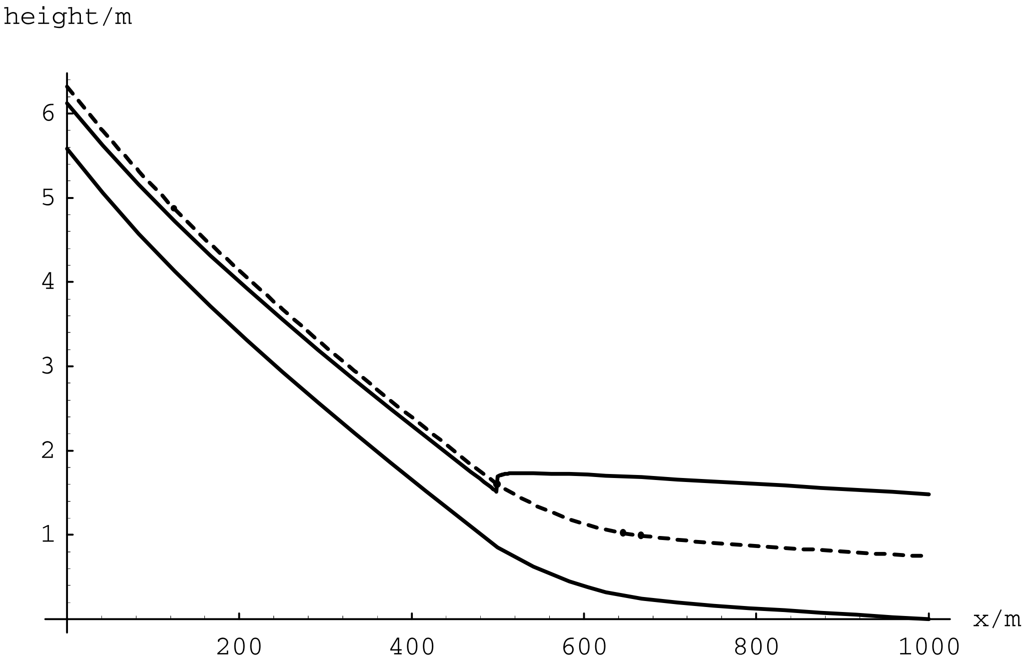

Figure 7: Stage and Bed Level for UR7

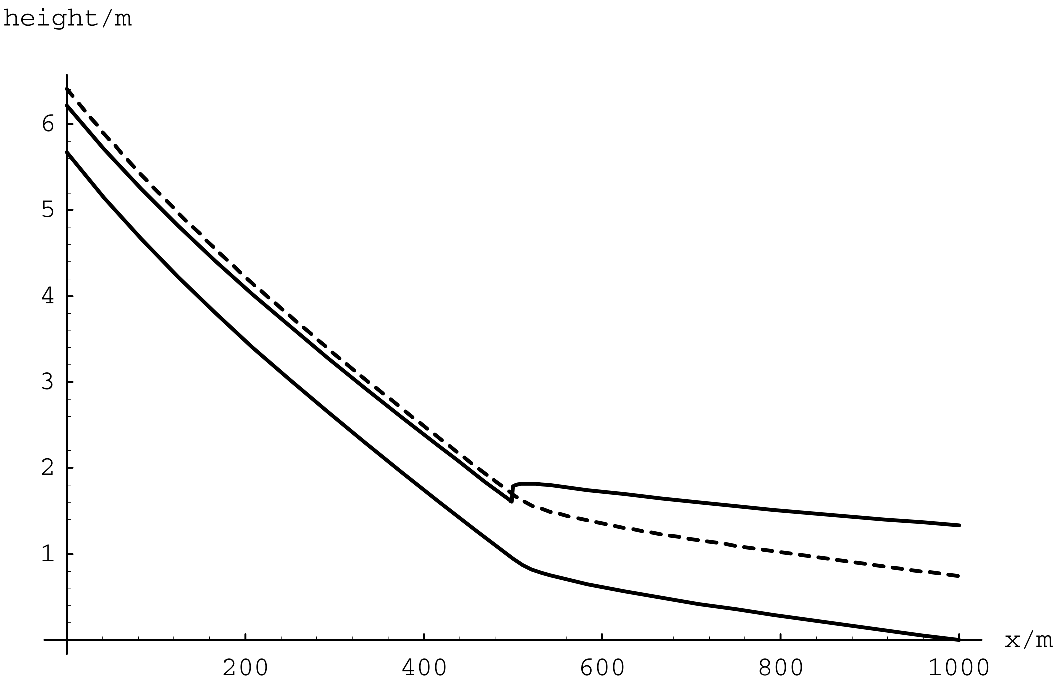

Figure 8: Stage and Bed Level for UR8

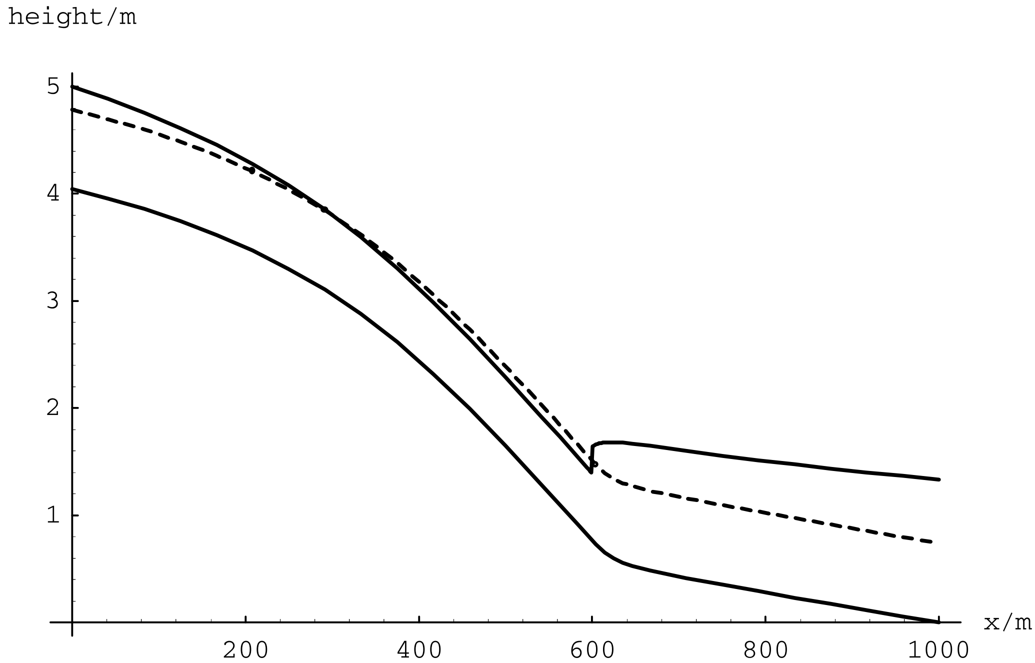

Figure 9: Stage and Bed Level for UR9

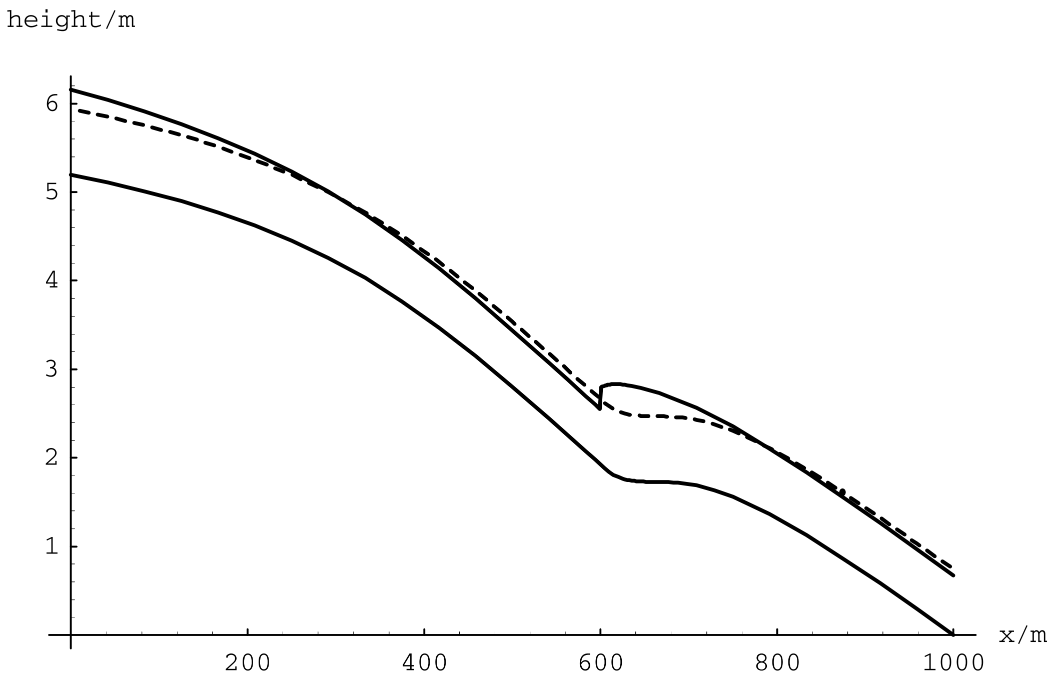

Figure 10: Stage and Bed Level for UR10

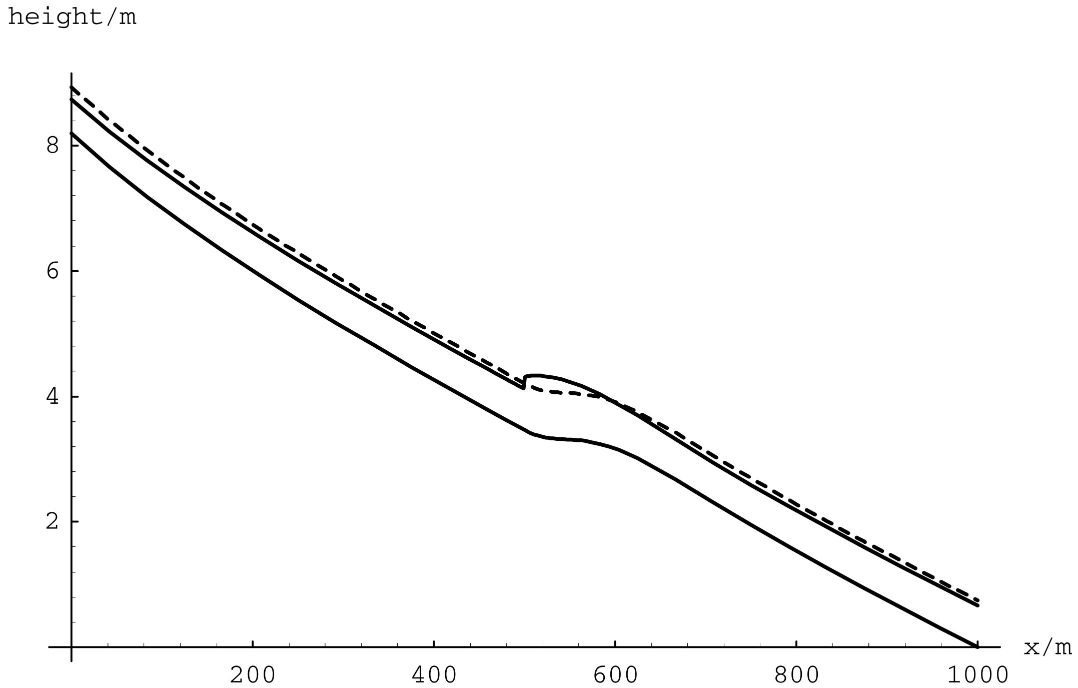

Figure 11: Stage and Bed Level for UR11

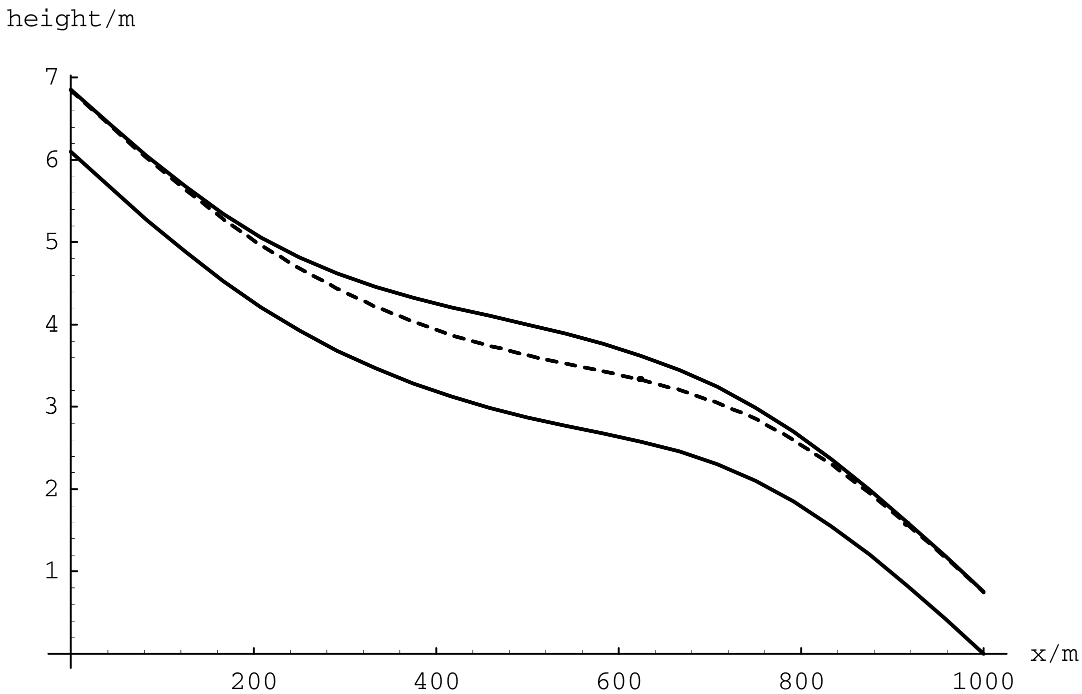

Figure 12: Stage and Bed Level for UT1

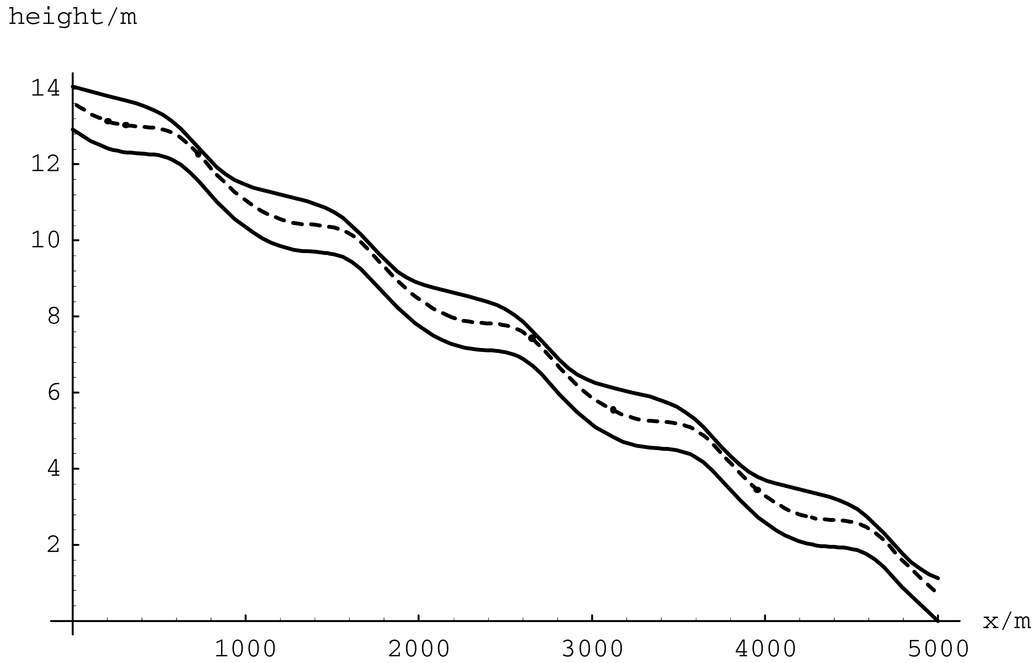

Figure 13: Stage and Bed Level for UT2

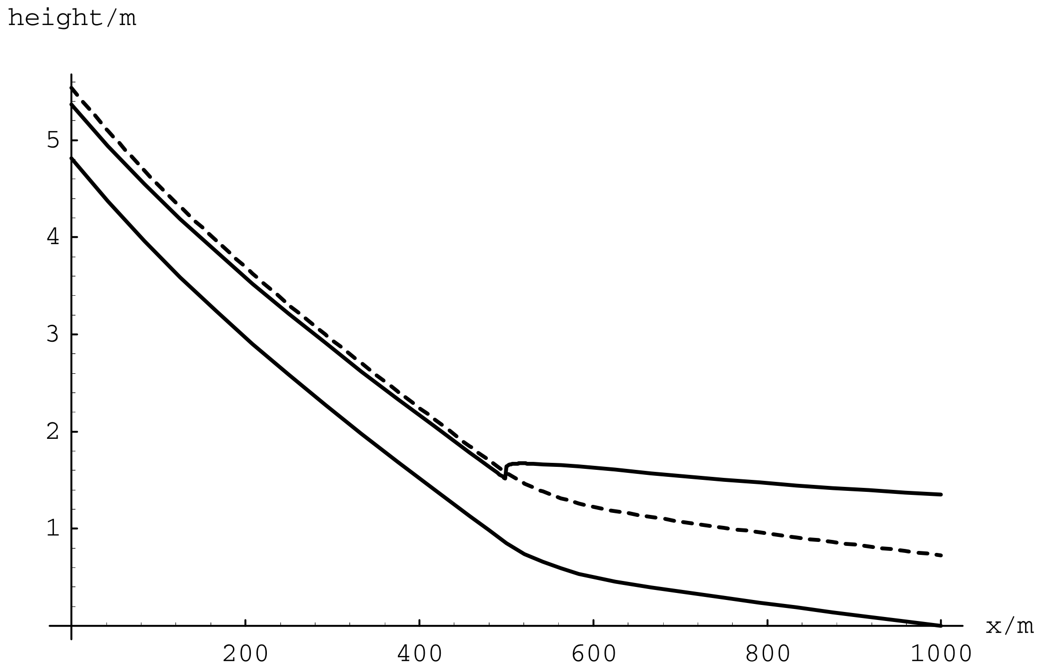

Figure 14: Stage and Bed Level for UT3

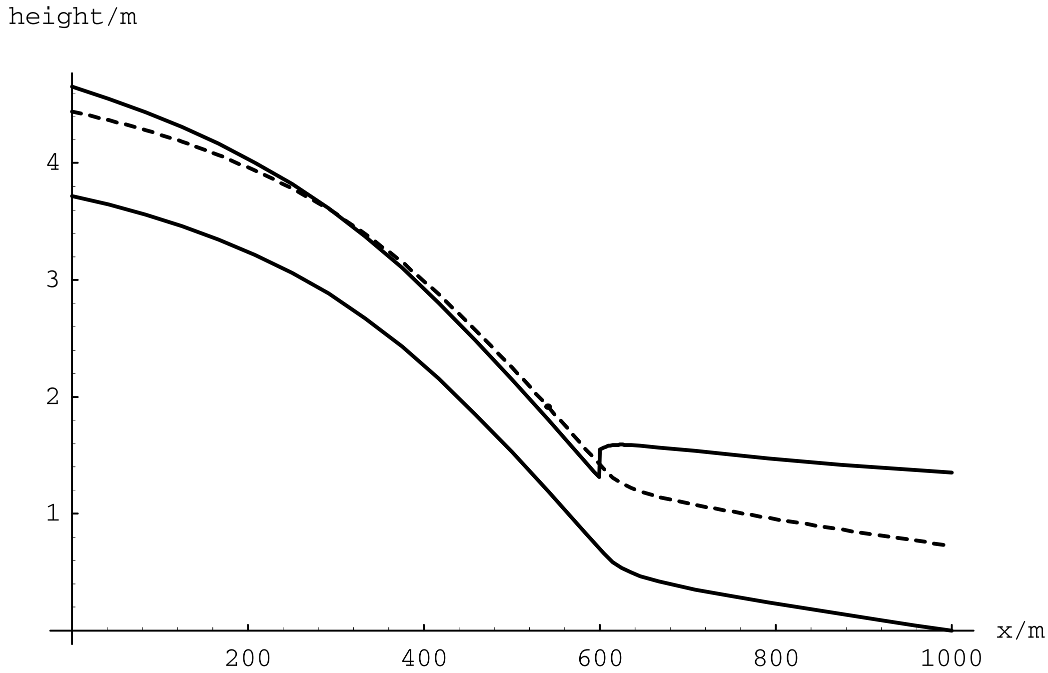

Figure 15: Stage and Bed Level for UT4

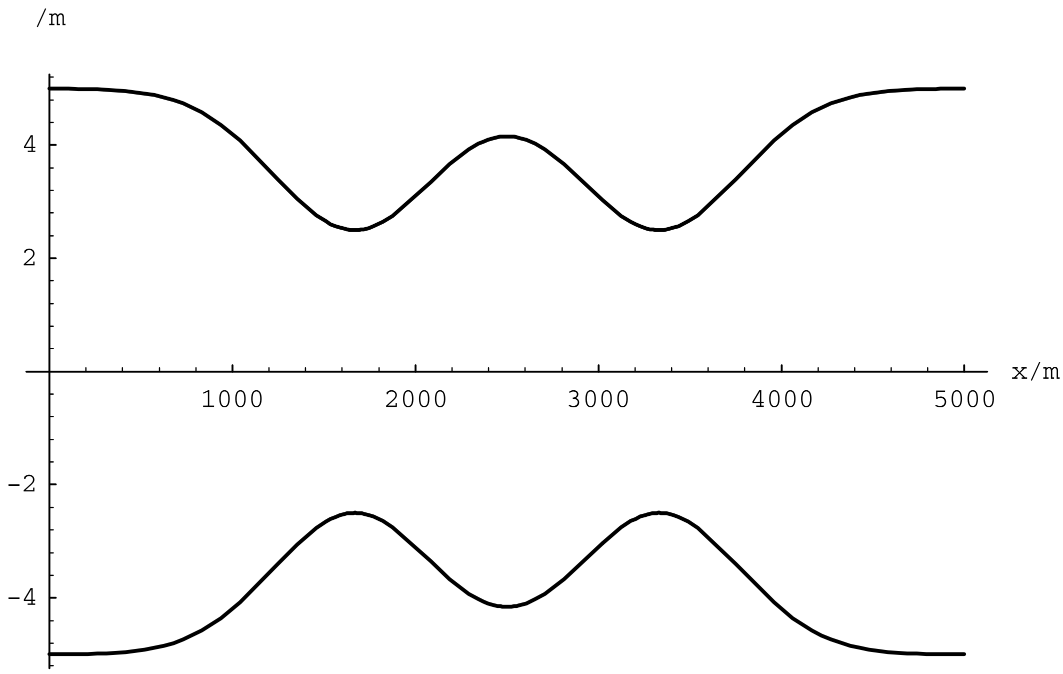

Figure 16: Channel Profile for VR1

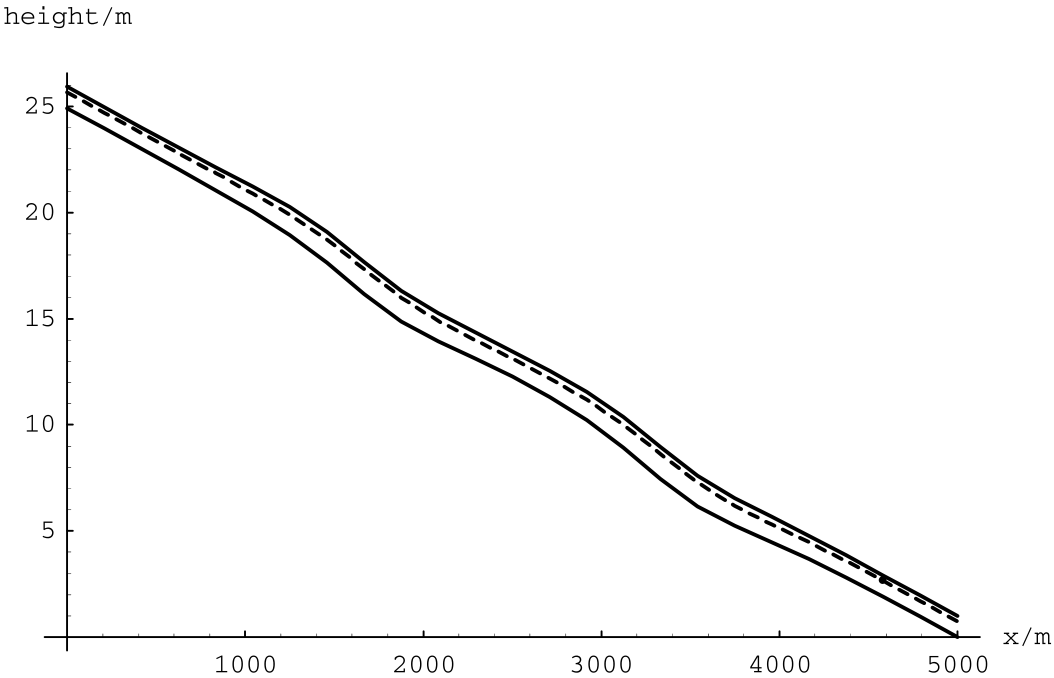

Figure 17: Stage and Bed Level for VR1

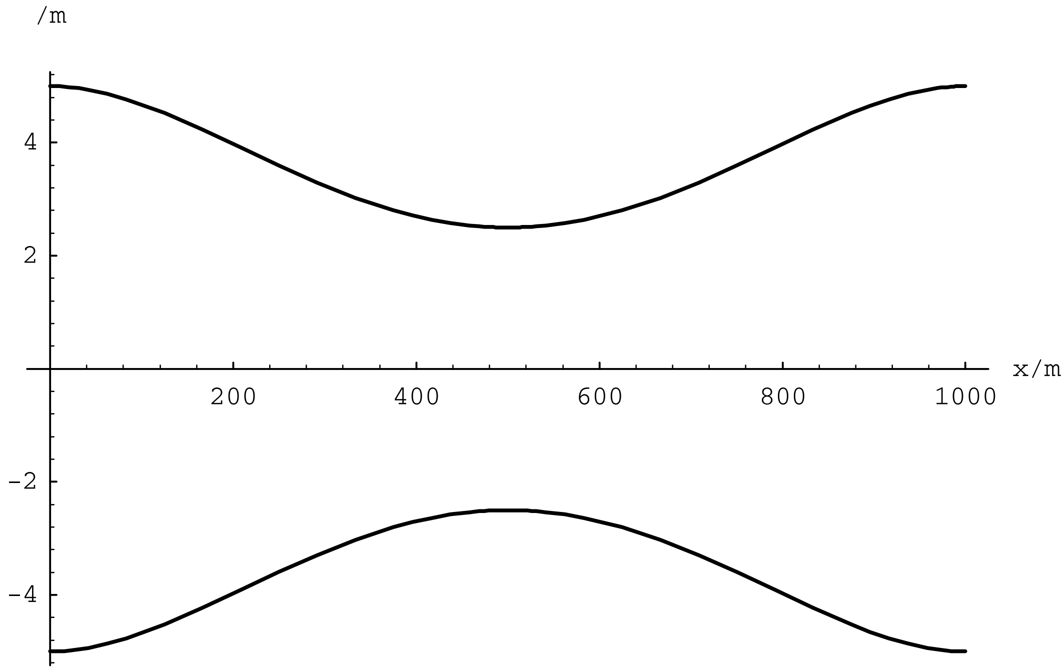

Figure 18: Channel Profile for VR2

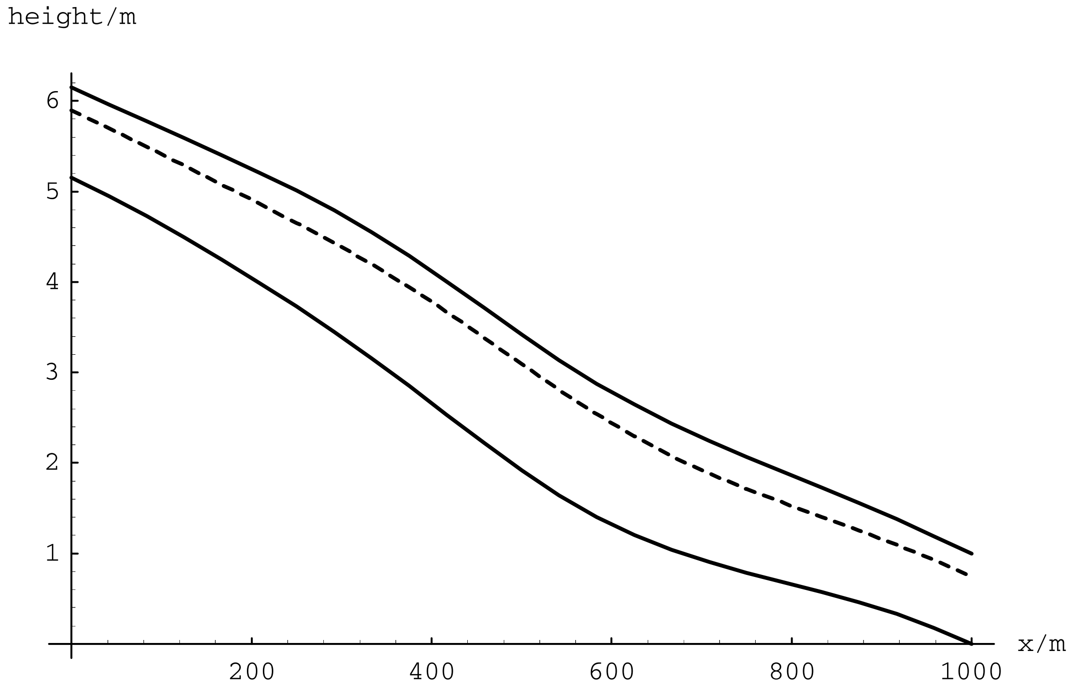

Figure 19: Stage and Bed Level for VR2

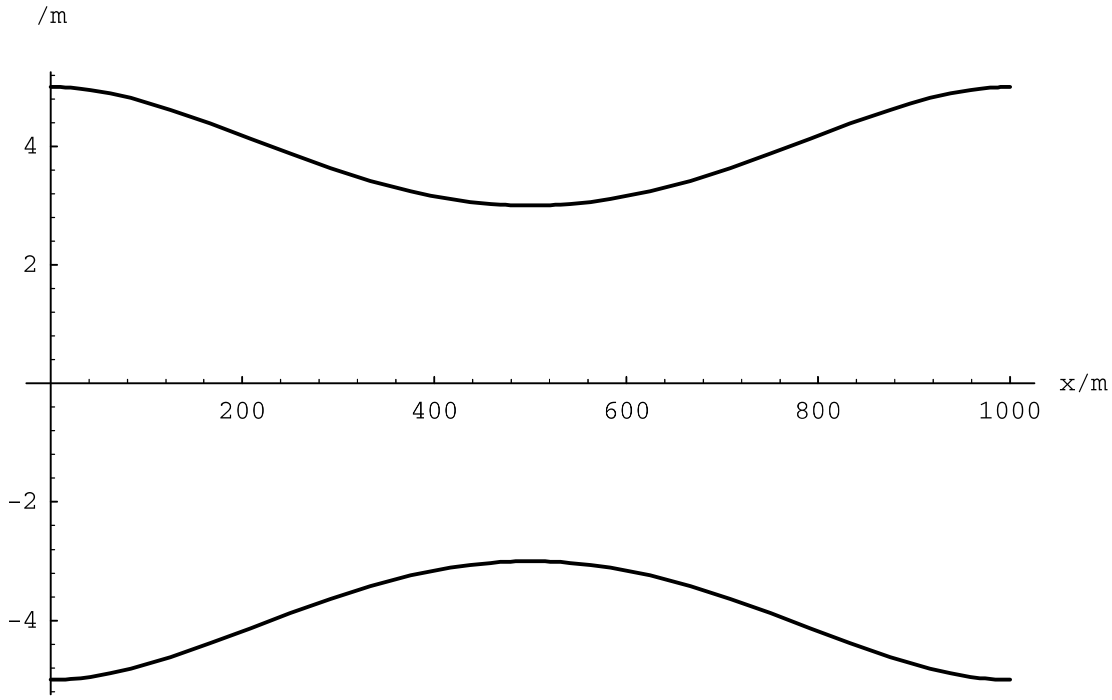

Figure 20: Channel Profile for VR3 and VR4

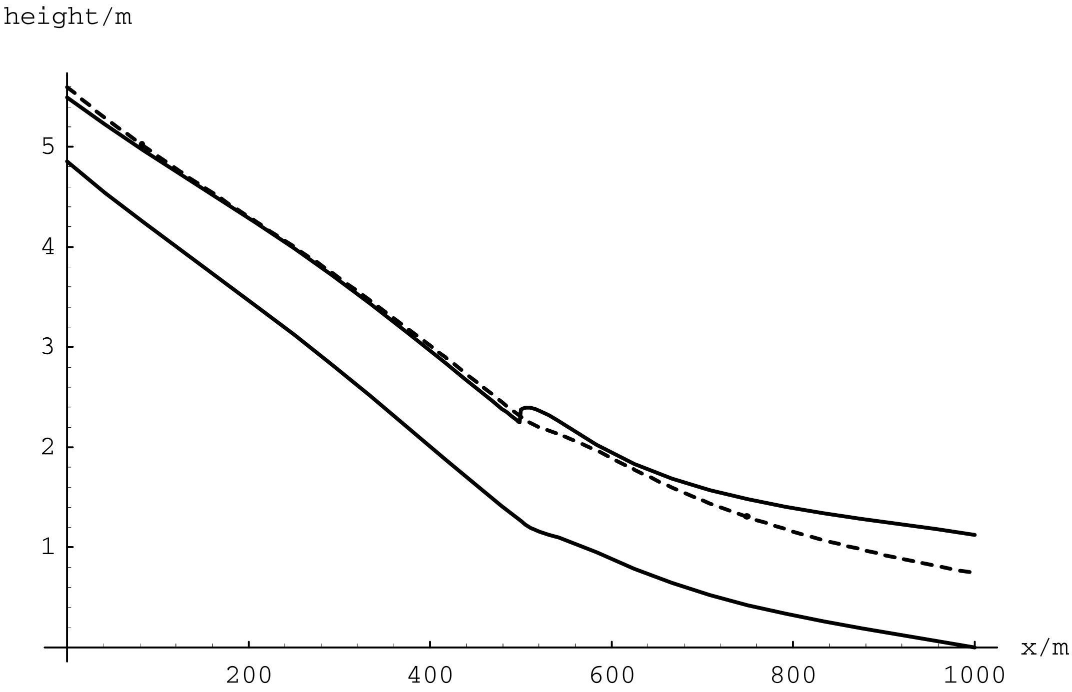

Figure 21: Stage and Bed Level for VR3

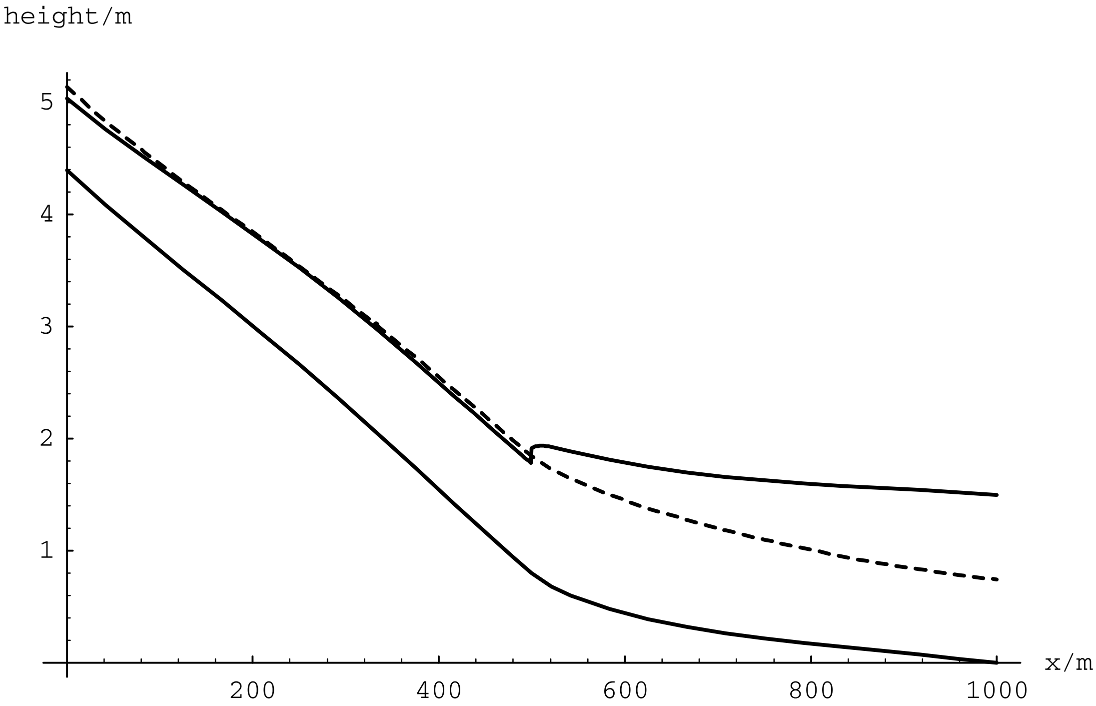

Figure 22: Stage and Bed Level for VR4

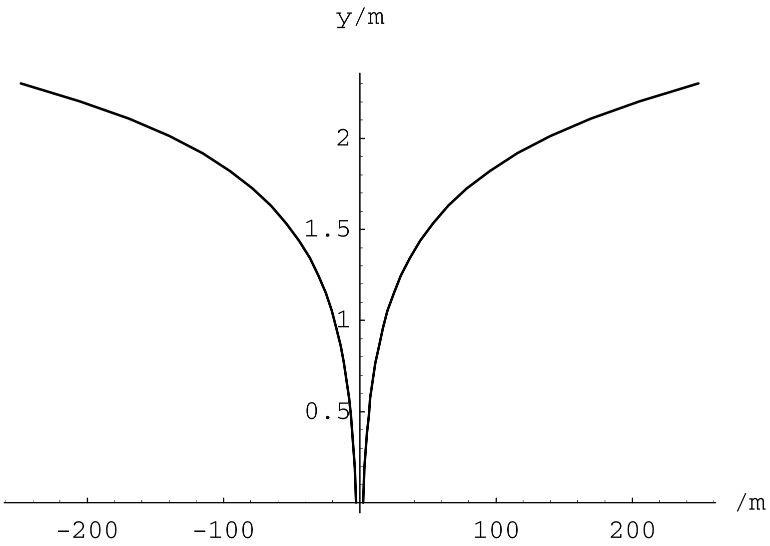

Figure 23: Channel Cross-Section for UE1, UE2 and UE3

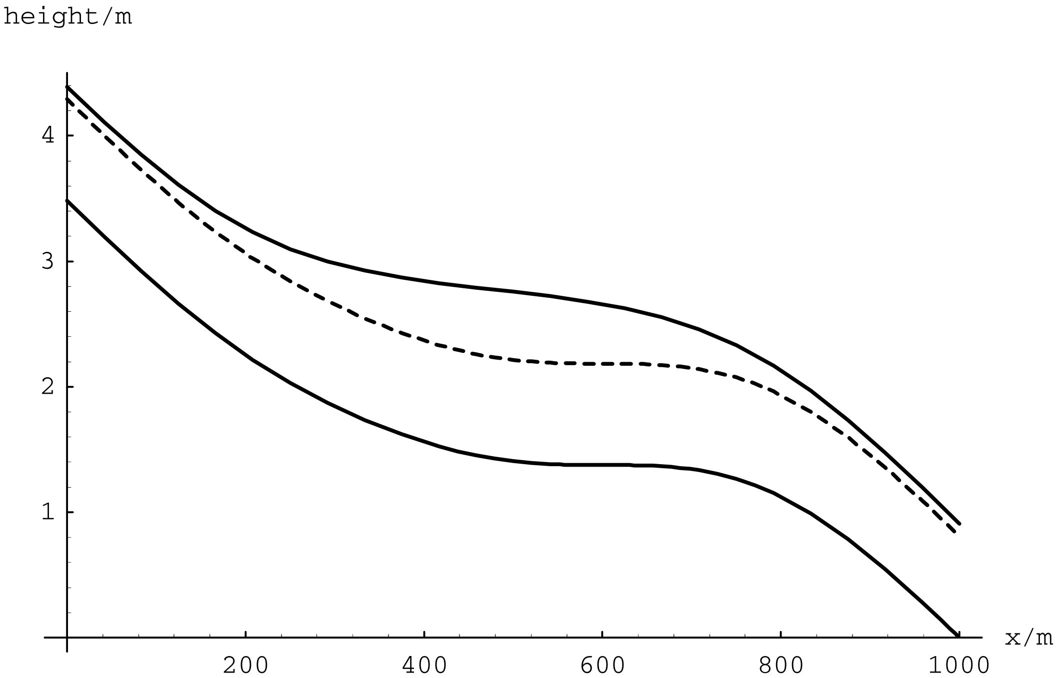

Figure 24: Stage and Bed Level for UE1

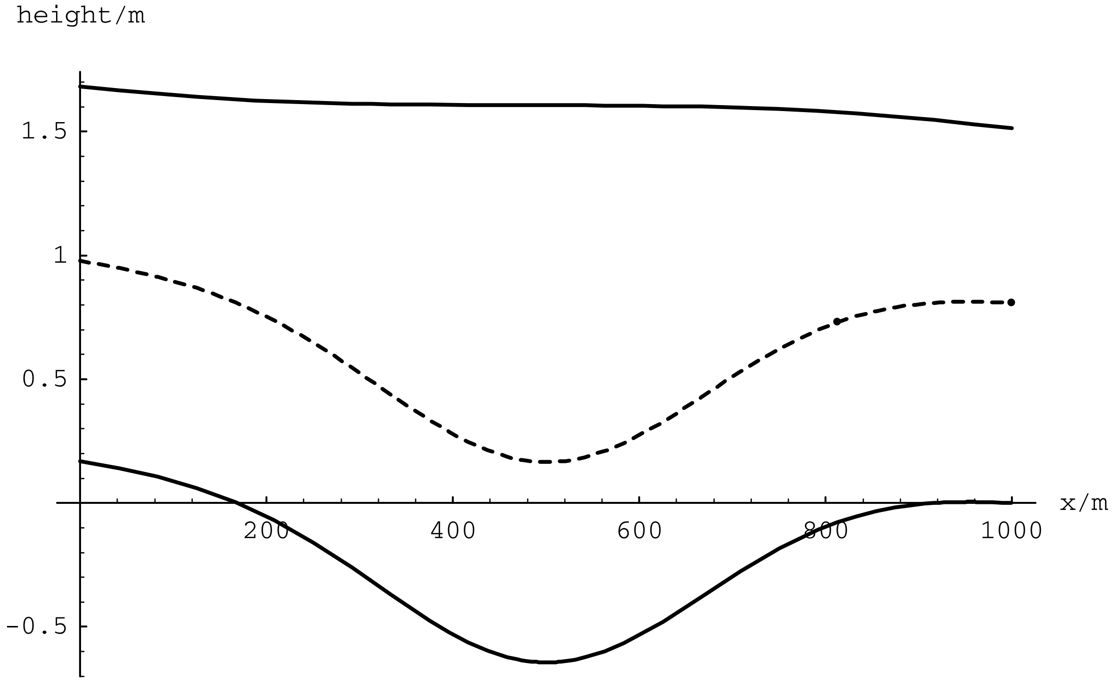

Figure 25: Stage and Bed Level for UE2

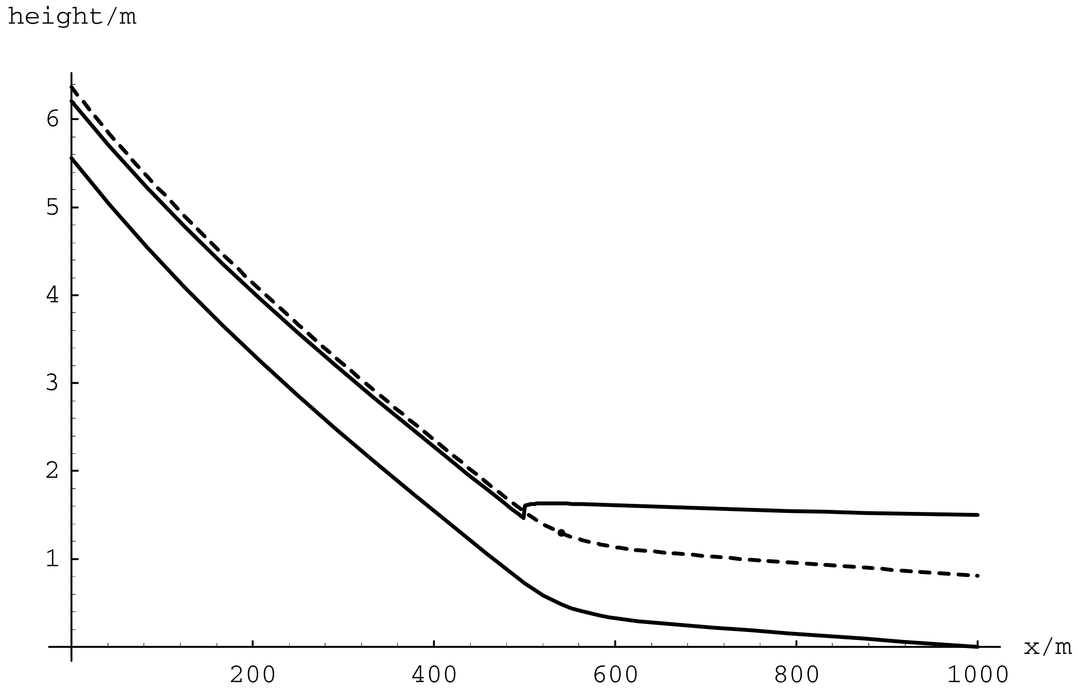

Figure 26: Stage and Bed Level for UE3

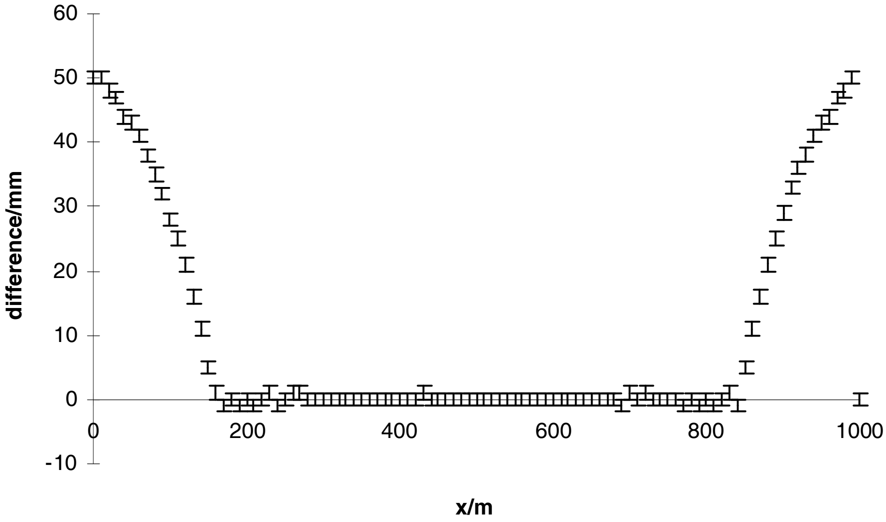

Figure 27: Difference between FLUCOMP and Analytic for UR1

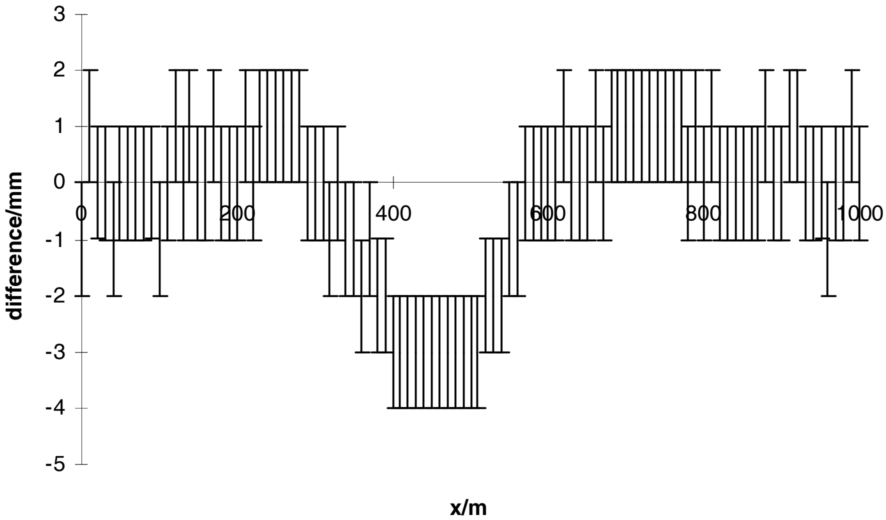

Figure 28: Difference between Osher and Analytic for UR1

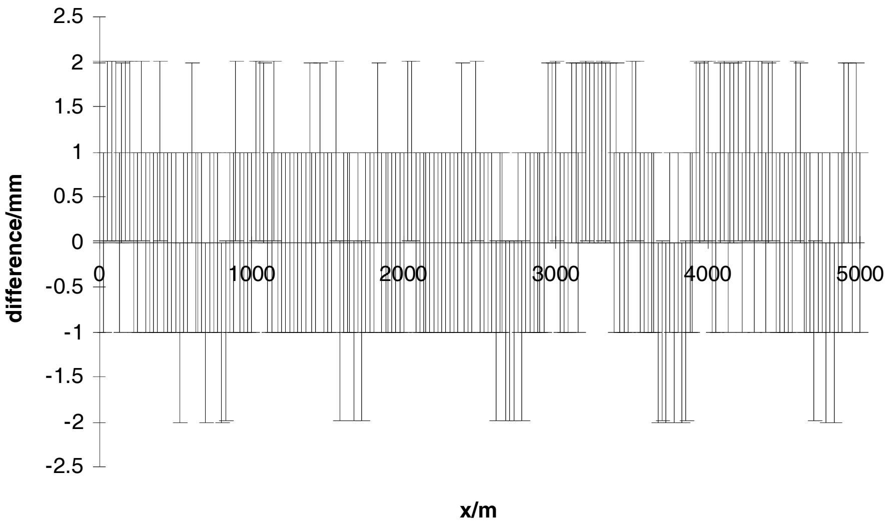

Figure 29: Difference between FLUCOMP and Analytic for UR3

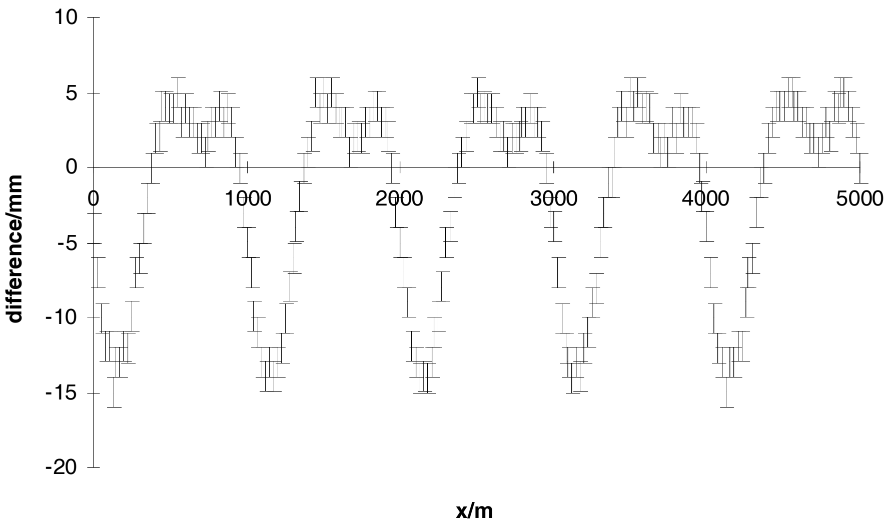

Figure 30: Difference between Osher and Analytic for UR3

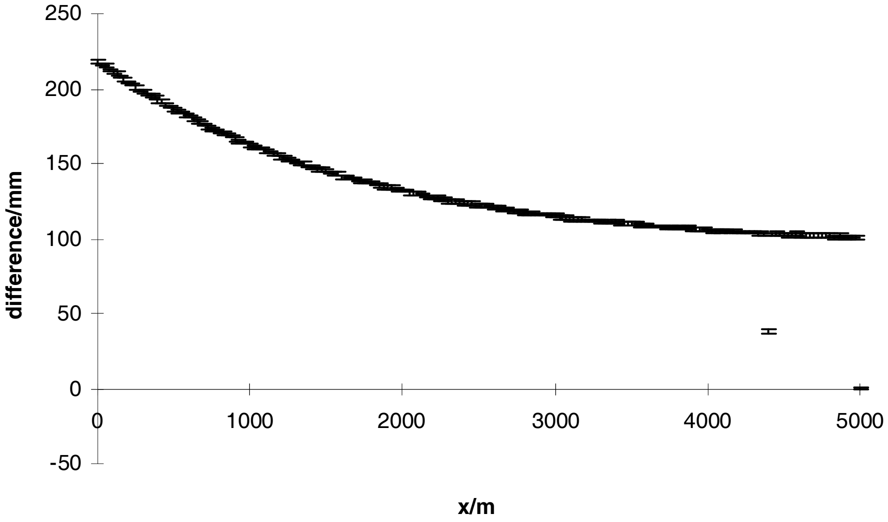

Figure 31: Difference between FLUCOMP and Analytic for UR5

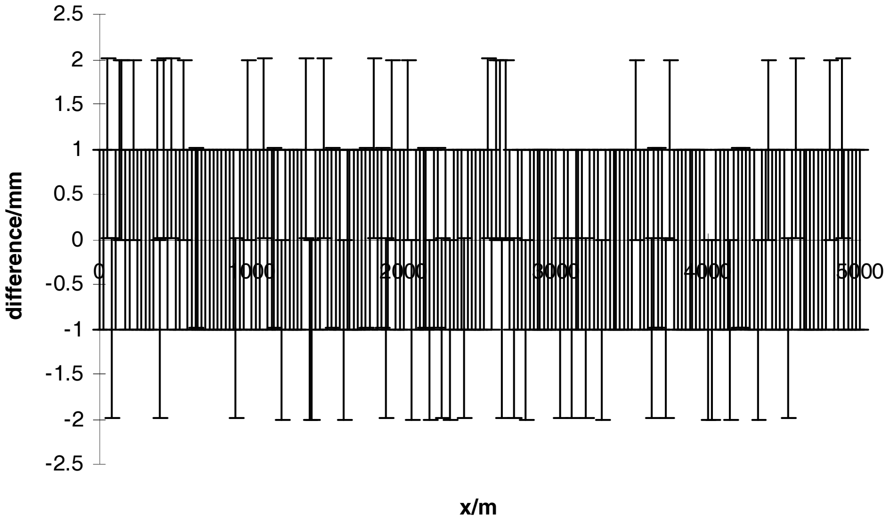

Figure 32: Difference between Osher and Analytic for UR5

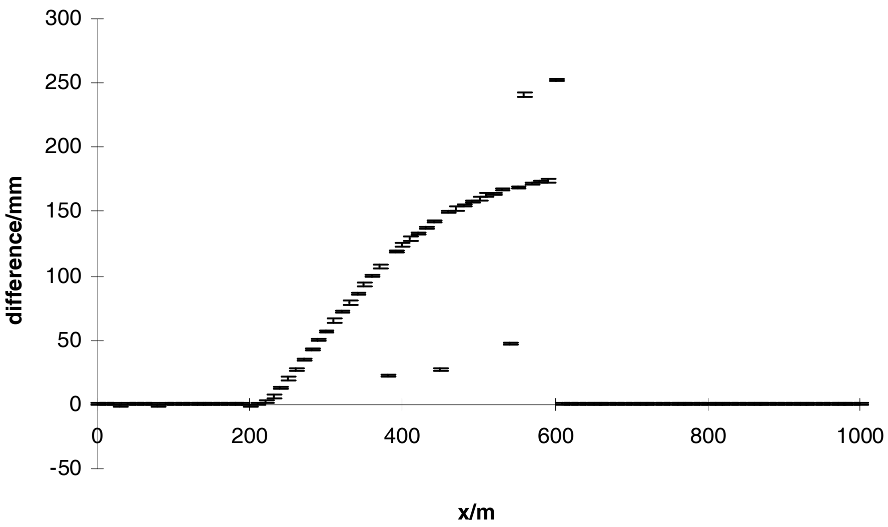

Figure 33: Difference between FLUCOMP and Analytic for UR9

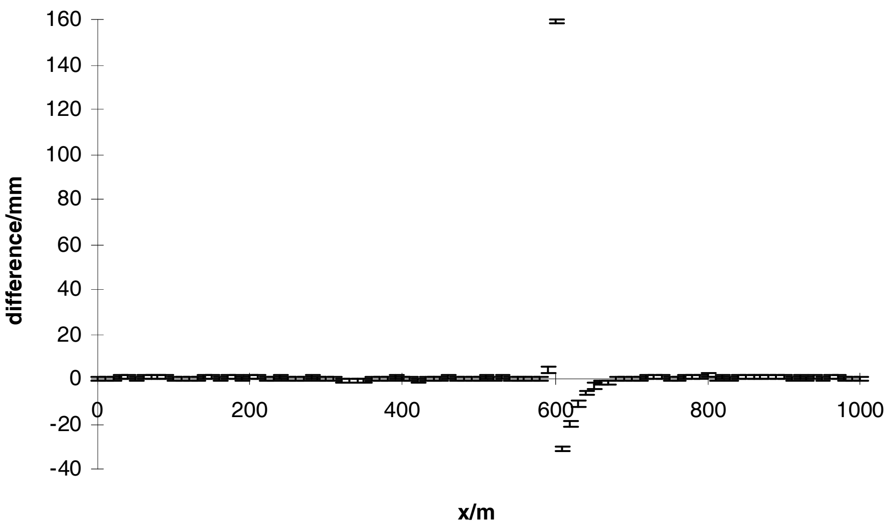

Figure 34: Difference between Osher and Analytic for UR9

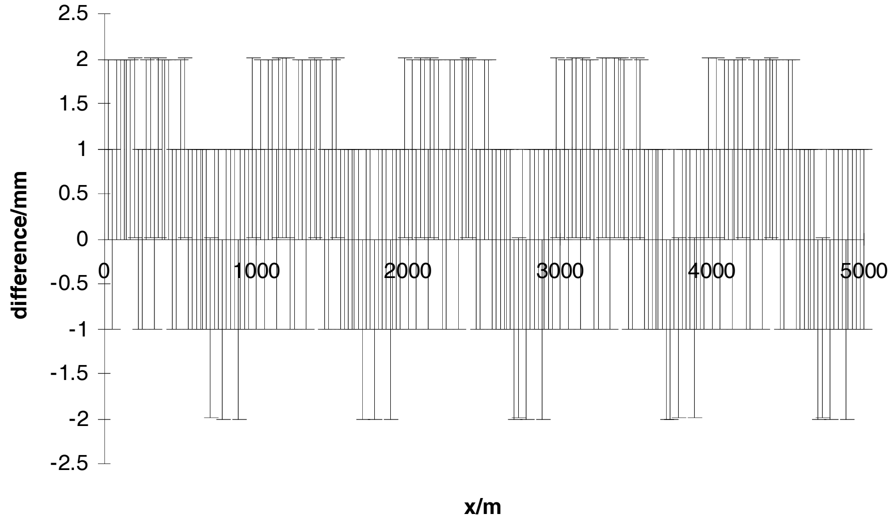

Figure 35: Difference between FLUCOMP and Analytic for UT2

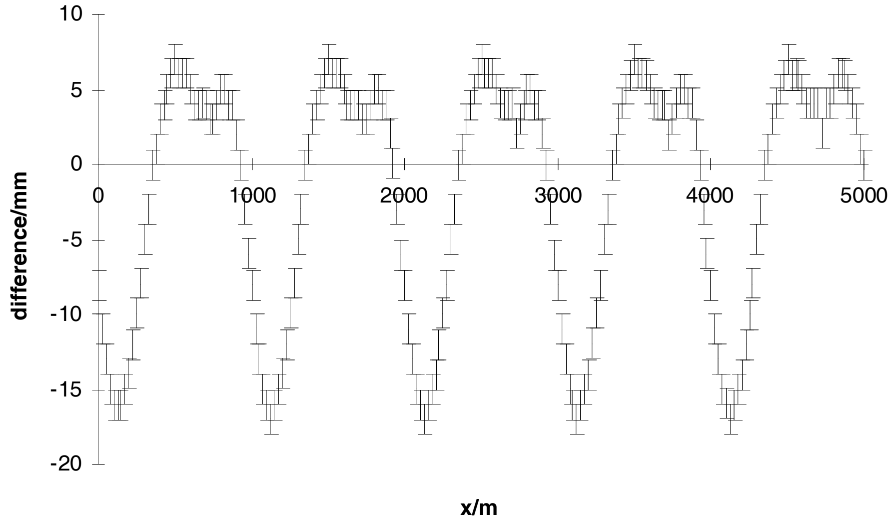

Figure 36: Difference between Osher and Analytic for UT2

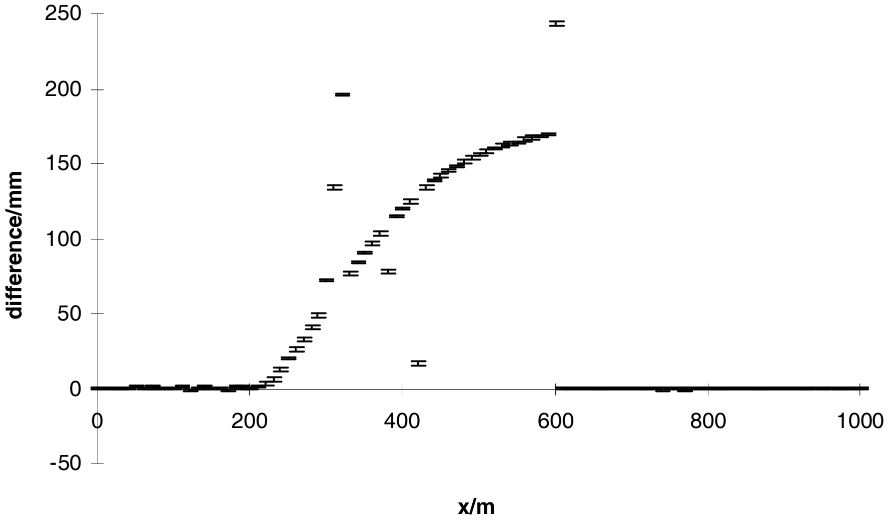

Figure 37: Difference between FLUCOMP and Analytic for UT4

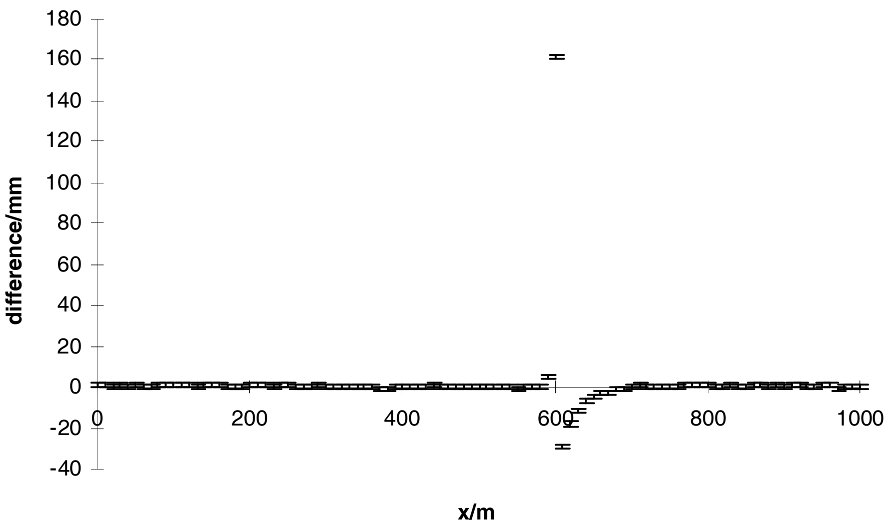

Figure 38: Difference between Osher and Analytic for UT4

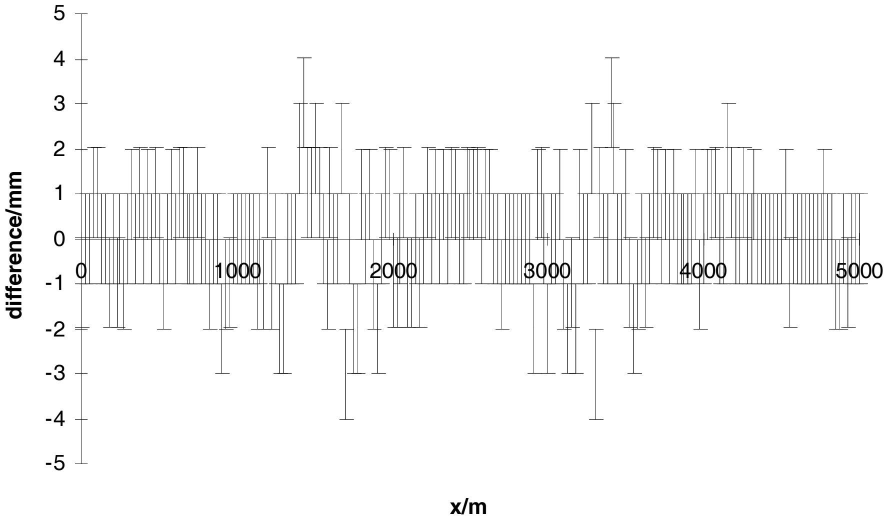

Figure 39: Difference between FLUCOMP and Analytic for VR1

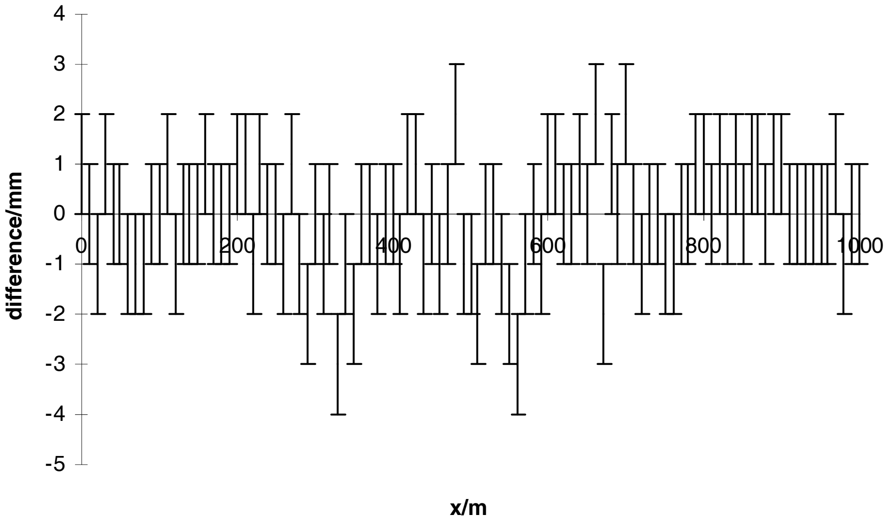

Figure 40: Difference between FLUCOMP and Analytic for VR2

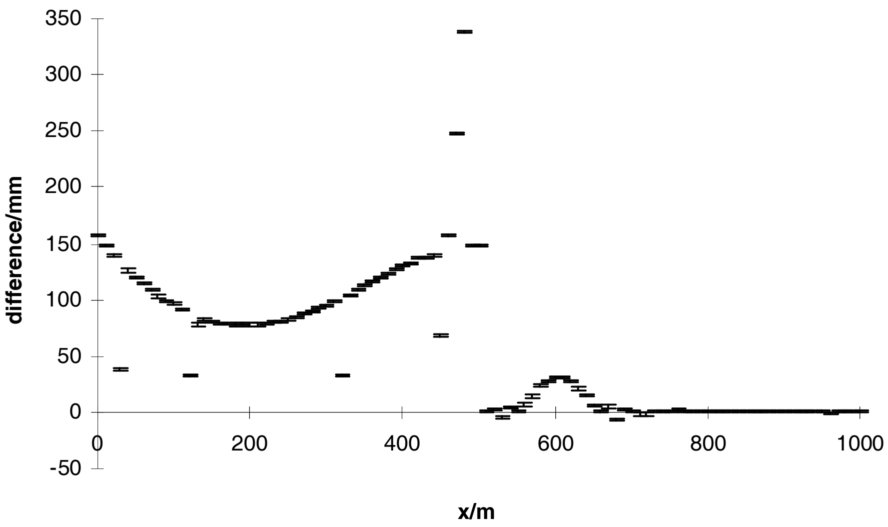

Figure 41: Difference between FLUCOMP and Analytic for VR3

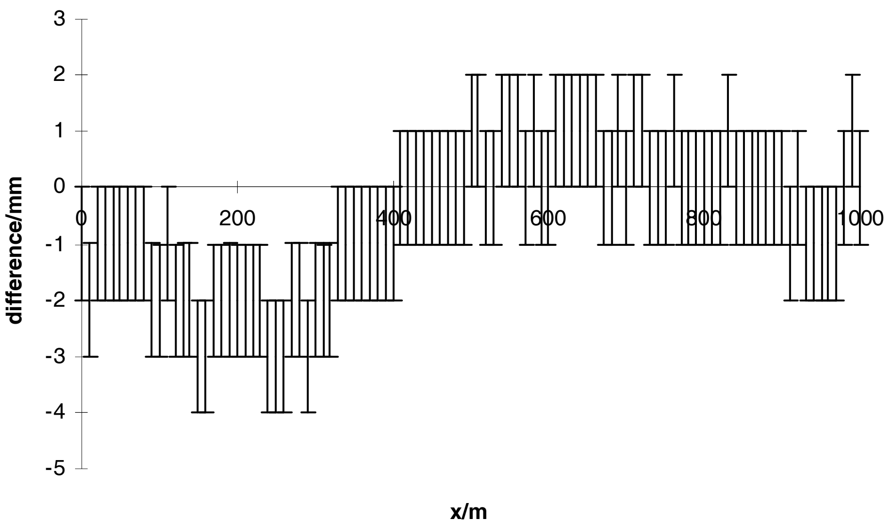

Figure 42: Difference between FLUCOMP and Analytic for UE1

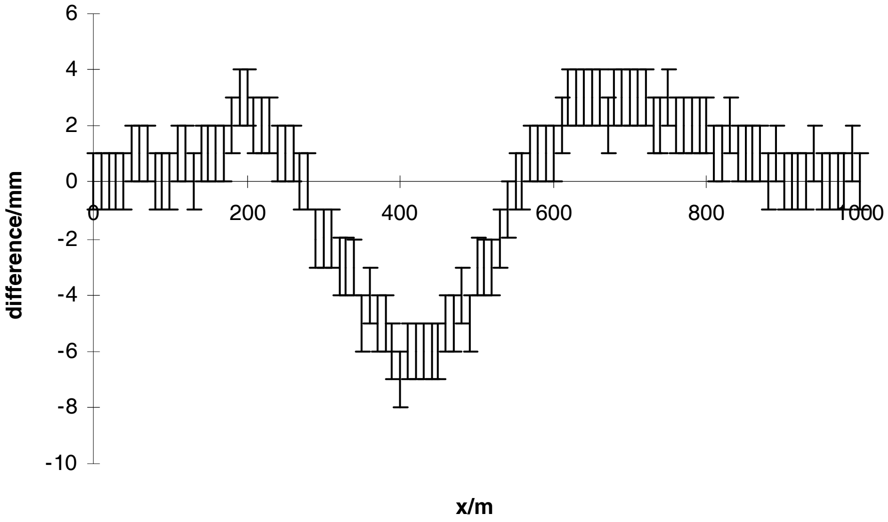

Figure 43: Difference between Osher and Analytic for UE1

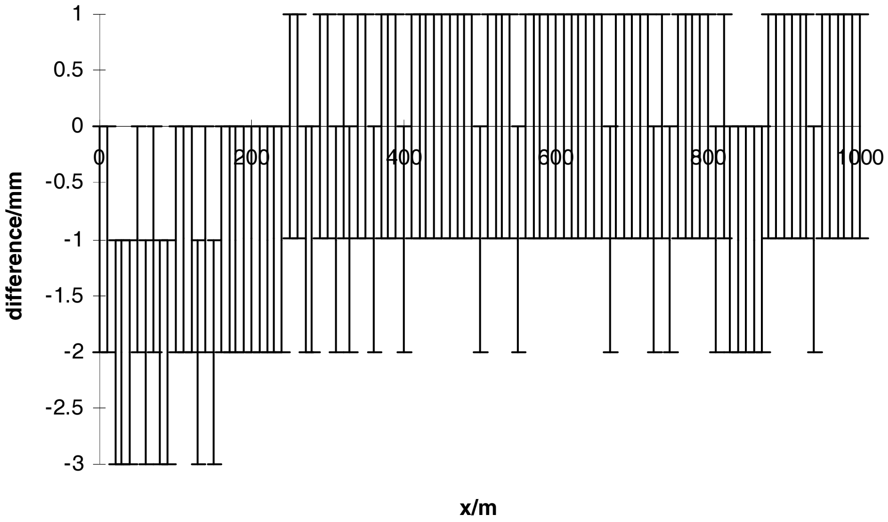

Figure 44 Difference between FLUCOMP and Analytic for UE2

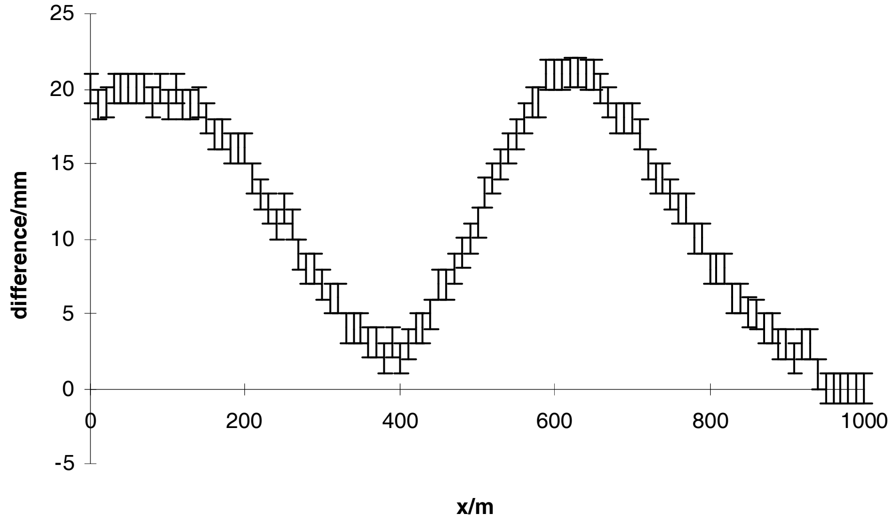

Figure 45: Difference between Osher and Analytic for UE2

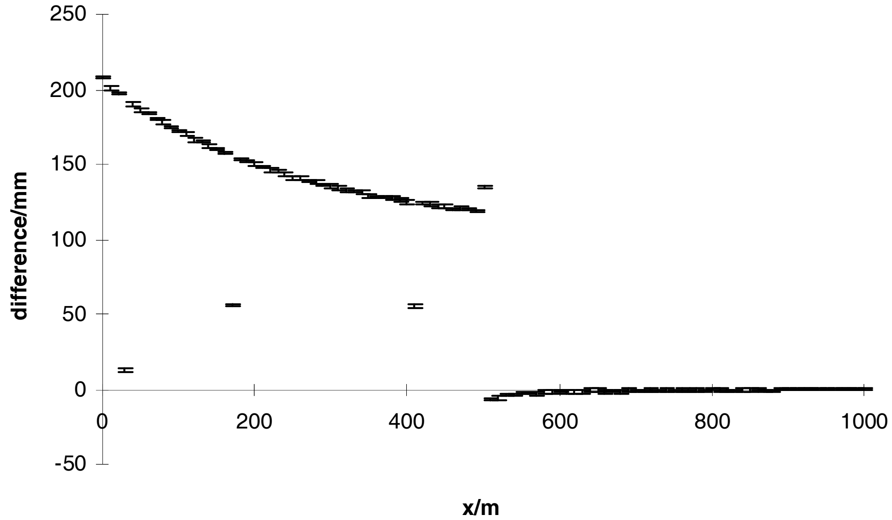

Figure 46: Difference between FLUCOMP and Analytic for UE3

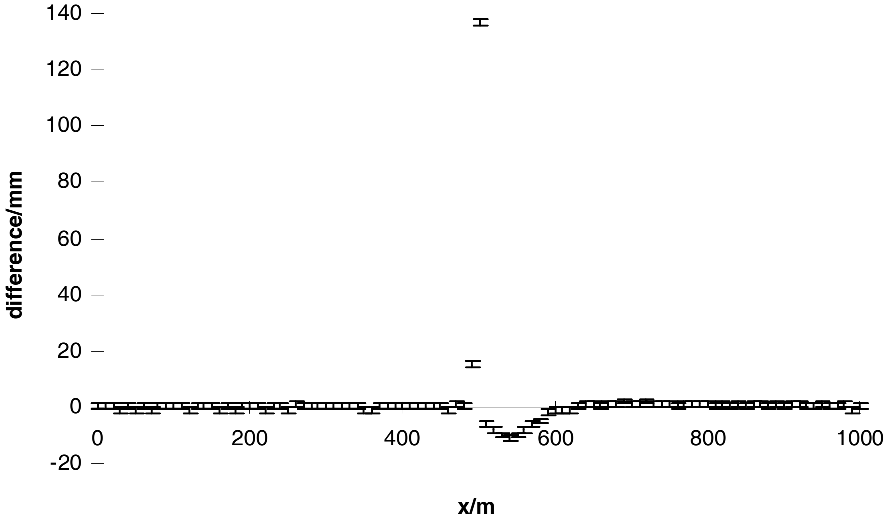

Figure 47: Difference between Osher and Analytic for UE3

TABLES

50	0.92E-04				
100.	37.29346	-56.22126	56.38843	-106.04945	178.82456
200.	37.29345	-56.22032	56.31741	-103.68202	149.23168
300.	37.29339	-56.21426	56.09042	-99.89895	125.58748
400.	37.29314	-56.19757	55.67305	-95.26151	106.26482
500.	37.29250	-56.16517	55.06555	-90.19897	90.44437
600.	37.29119	-56.11293	54.28202	-84.97543	77.38553
700.	37.28894	-56.03801	53.34549	-79.77249	66.54607
800.	37.28546	-55.93867	52.28112	-74.70406	57.49529
900.	37.28049	-55.81418	51.11406	-69.84131	49.89724
1000.	37.27375	-55.66462	49.86766	-65.22502	43.48573
1100.	37.26506	-55.49064	48.56285	-60.87567	38.04904
1200.	37.25421	-55.29338	47.21787	-56.79997	33.41757
1300.	37.24106	-55.07424	45.84829	-52.99558	29.45466
1400.	37.22550	-54.83485	44.46719	-49.45429	26.04958
1500.	37.20744	-54.57692	43.08540	-46.16432	23.11210
1600.	37.18684	-54.30220	41.71178	-43.11184	20.56837
1700.	37.16366	-54.01242	40.35347	-40.28201	18.35757
1800.	37.13789	-53.70929	39.01612	-37.65976	16.42944
1900.	37.10955	-53.39442	37.70416	-35.23021	14.74225
2000.	37.07867	-53.06934	36.42096	-32.97898	13.26118
2100.	37.04528	-52.73549	35.16902	-30.89241	11.95707
2200.	37.00945	-52.39420	33.95012	-28.95764	10.80543
2300.	36.97122	-52.04669	32.76544	-27.16268	9.78556
2400.	36.93068	-51.69410	31.61569	-25.49637	8.87996
2500.	36.88788	-51.33745	30.50116	-23.94842	8.07373
2600.	36.84291	-50.97769	29.42186	-22.50934	7.35419
2700.	36.79584	-50.61565	28.37751	-21.17044	6.71049
2800.	36.74676	-50.25210	27.36766	-19.92371	6.13330
2900.	36.69575	-49.88774	26.39170	-18.76186	5.61462
3000.	36.64289	-49.52319	25.44891	-17.67818	5.14752
3100.	36.58827	-49.15901	24.53845	-16.66657	4.72601
3200.	36.53195	-48.79570	23.65947	-15.72142	4.34490
3300.	36.47403	-48.43370	22.81104	-14.83764	3.99968
3400.	36.41459	-48.07342	21.99222	-14.01055	3.68638
3500.	36.35369	-47.71522	21.20206	-13.23588	3.40158
3600.	36.29142	-47.35940	20.43960	-12.50974	3.14224
3700.	36.22786	-47.00627	19.70391	-11.82853	2.90571
3800.	36.16307	-46.65607	18.99403	-11.18901	2.68966
3900.	36.09713	-46.30902	18.30908	-10.58817	2.49201
4000.	36.03011	-45.96533	17.64814	-10.02327	2.31095
4100.	35.96208	-45.62518	17.01036	-9.49178	2.14487
4200.	35.89311	-45.28873	16.39489	-8.99140	1.99231
4300.	35.82326	-44.95611	15.80093	-8.52001	1.85201
4400.	35.75260	-44.62746	15.22770	-8.07564	1.72284
4500.	35.68119	-44.30288	14.67444	-7.65650	1.60377
4600.	35.60910	-43.98247	14.14043	-7.26094	1.49389
4700.	35.53639	-43.66632	13.62496	-6.88741	1.39238
4800.	35.46312	-43.35455	13.12746	-6.53457	1.29854
4900.	35.38931	-43.04698	12.64688	-6.20084	1.21163
5000.	35.31524	-42.74469	12.18420	-5.88609	1.13134

Table 1: Coefficients for bed level for UR5

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	7.084	7.833	510.	3.345	4.457
10.	6.971	7.720	520.	3.313	4.423
20.	6.857	7.608	530.	3.282	4.389
30.	6.745	7.497	540.	3.251	4.354
40.	6.633	7.388	550.	3.220	4.318
50.	6.523	7.279	560.	3.190	4.282
60.	6.413	7.171	570.	3.160	4.244
70.	6.304	7.065	580.	3.129	4.205
80.	6.196	6.960	590.	3.098	4.165
90.	6.089	6.856	600.	3.066	4.124
100.	5.984	6.755	610.	3.034	4.081
110.	5.881	6.655	620.	3.001	4.037
120.	5.779	6.557	630.	2.966	3.991
130.	5.678	6.461	640.	2.930	3.943
140.	5.580	6.368	650.	2.892	3.893
150.	5.483	6.277	660.	2.853	3.841
160.	5.388	6.188	670.	2.811	3.787
170.	5.296	6.102	680.	2.768	3.730
180.	5.205	6.019	690.	2.722	3.672
190.	5.117	5.939	700.	2.673	3.610
200.	5.031	5.861	710.	2.622	3.547
210.	4.948	5.786	720.	2.568	3.480
220.	4.866	5.714	730.	2.511	3.412
230.	4.787	5.644	740.	2.451	3.340
240.	4.711	5.578	750.	2.388	3.266
250.	4.636	5.514	760.	2.322	3.189
260.	4.564	5.453	770.	2.253	3.110
270.	4.494	5.395	780.	2.180	3.028
280.	4.427	5.339	790.	2.105	2.943
290.	4.361	5.286	800.	2.026	2.856
300.	4.297	5.235	810.	1.945	2.766
310.	4.236	5.186	820.	1.860	2.674
320.	4.176	5.139	830.	1.773	2.580
330.	4.119	5.094	840.	1.684	2.484
340.	4.063	5.051	850.	1.591	2.385
350.	4.009	5.009	860.	1.497	2.285
360.	3.956	4.969	870.	1.400	2.183
370.	3.906	4.930	880.	1.301	2.079
380.	3.857	4.893	890.	1.200	1.974
390.	3.809	4.856	900.	1.097	1.867
400.	3.763	4.821	910.	0.993	1.759
410.	3.719	4.786	920.	0.887	1.650
420.	3.676	4.752	930.	0.779	1.540
430.	3.634	4.718	940.	0.671	1.429
440.	3.594	4.685	950.	0.561	1.317
450.	3.555	4.653	960.	0.450	1.205
460.	3.517	4.620	970.	0.339	1.091
470.	3.481	4.588	980.	0.227	0.978
480.	3.445	4.555	990.	0.114	0.863
490.	3.411	4.523	1000.	0.000	0.748
500.	3.377	4.490			

Table 2: Bed level and stage data for UR1

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	19.394	20.506	500.	18.269	19.382
10.	19.351	20.479	510.	18.246	19.343
20.	19.308	20.452	520.	18.222	19.303
30.	19.268	20.426	530.	18.195	19.261
40.	19.228	20.402	540.	18.167	19.218
50.	19.189	20.378	550.	18.136	19.172
60.	19.152	20.355	560.	18.103	19.125
70.	19.115	20.333	570.	18.068	19.075
80.	19.080	20.311	580.	18.030	19.024
90.	19.046	20.291	590.	17.990	18.970
100.	19.013	20.270	600.	17.947	18.914
110.	18.981	20.251	610.	17.902	18.857
120.	18.950	20.231	620.	17.854	18.797
130.	18.920	20.212	630.	17.803	18.736
140.	18.891	20.194	640.	17.750	18.672
150.	18.863	20.175	650.	17.695	18.607
160.	18.836	20.157	660.	17.637	18.540
170.	18.810	20.139	670.	17.576	18.472
180.	18.785	20.122	680.	17.514	18.402
190.	18.762	20.104	690.	17.449	18.332
200.	18.739	20.086	700.	17.383	18.260
210.	18.717	20.069	710.	17.315	18.188
220.	18.696	20.051	720.	17.245	18.115
230.	18.676	20.034	730.	17.175	18.042
240.	18.657	20.016	740.	17.104	17.969
250.	18.639	19.999	750.	17.032	17.897
260.	18.621	19.981	760.	16.959	17.825
270.	18.605	19.963	770.	16.887	17.754
280.	18.589	19.944	780.	16.815	17.684
290.	18.574	19.926	790.	16.743	17.616
300.	18.559	19.907	800.	16.672	17.549
310.	18.545	19.887	810.	16.601	17.484
320.	18.532	19.868	820.	16.531	17.420
330.	18.518	19.847	830.	16.463	17.359
340.	18.506	19.827	840.	16.395	17.299
350.	18.493	19.806	850.	16.329	17.242
360.	18.481	19.784	860.	16.265	17.187
370.	18.469	19.761	870.	16.202	17.134
380.	18.457	19.738	880.	16.140	17.083
390.	18.444	19.714	890.	16.080	17.035
400.	18.432	19.689	900.	16.021	16.988
410.	18.419	19.664	910.	15.964	16.944
420.	18.406	19.637	920.	15.909	16.902
430.	18.392	19.610	930.	15.855	16.862
440.	18.378	19.581	940.	15.802	16.823
450.	18.362	19.551	950.	15.751	16.787
460.	18.346	19.520	960.	15.701	16.752
470.	18.329	19.488	970.	15.653	16.719
480.	18.310	19.454	980.	15.605	16.687
490.	18.291	19.419	990.	15.560	16.657

Table 3: Bed level and stage data for UR3

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
1000.	15.515	16.628	1500.	14.391	15.503
1010.	15.472	16.600	1510.	14.368	15.465
1020.	15.430	16.573	1520.	14.343	15.424
1030.	15.389	16.548	1530.	14.316	15.383
1040.	15.349	16.523	1540.	14.288	15.339
1050.	15.310	16.499	1550.	14.257	15.293
1060.	15.273	16.476	1560.	14.224	15.246
1070.	15.236	16.454	1570.	14.189	15.196
1080.	15.201	16.433	1580.	14.151	15.145
1090.	15.167	16.412	1590.	14.111	15.091
1100.	15.134	16.392	1600.	14.068	15.036
1110.	15.102	16.372	1610.	14.023	14.978
1120.	15.071	16.352	1620.	13.975	14.918
1130.	15.041	16.333	1630.	13.925	14.857
1140.	15.012	16.315	1640.	13.871	14.793
1150.	14.984	16.296	1650.	13.816	14.728
1160.	14.957	16.278	1660.	13.758	14.661
1170.	14.931	16.261	1670.	13.697	14.593
1180.	14.907	16.243	1680.	13.635	14.524
1190.	14.883	16.225	1690.	13.570	14.453
1200.	14.860	16.208	1700.	13.504	14.381
1210.	14.838	16.190	1710.	13.436	14.309
1220.	14.817	16.173	1720.	13.367	14.236
1230.	14.797	16.155	1730.	13.296	14.163
1240.	14.778	16.137	1740.	13.225	14.091
1250.	14.760	16.120	1750.	13.153	14.018
1260.	14.743	16.102	1760.	13.081	13.946
1270.	14.726	16.084	1770.	13.008	13.875
1280.	14.710	16.065	1780.	12.936	13.806
1290.	14.695	16.047	1790.	12.864	13.737
1300.	14.680	16.028	1800.	12.793	13.670
1310.	14.666	16.009	1810.	12.722	13.605
1320.	14.653	15.989	1820.	12.653	13.541
1330.	14.640	15.969	1830.	12.584	13.480
1340.	14.627	15.948	1840.	12.517	13.420
1350.	14.614	15.927	1850.	12.451	13.363
1360.	14.602	15.905	1860.	12.386	13.308
1370.	14.590	15.883	1870.	12.323	13.255
1380.	14.578	15.859	1880.	12.261	13.204
1390.	14.565	15.835	1890.	12.201	13.156
1400.	14.553	15.811	1900.	12.143	13.110
1410.	14.540	15.785	1910.	12.085	13.065
1420.	14.527	15.758	1920.	12.030	13.023
1430.	14.513	15.731	1930.	11.976	12.983
1440.	14.499	15.702	1940.	11.923	12.945
1450.	14.484	15.672	1950.	11.872	12.908
1460.	14.467	15.641	1960.	11.822	12.873
1470.	14.450	15.609	1970.	11.774	12.840
1480.	14.432	15.575	1980.	11.727	12.808
1490.	14.412	15.540	1990.	11.681	12.778

Table 3 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
2000.	11.636	12.749	2500.	10.512	11.624
2010.	11.593	12.721	2510.	10.489	11.586
2020.	11.551	12.694	2520.	10.464	11.546
2030.	11.510	12.669	2530.	10.438	11.504
2040.	11.470	12.644	2540.	10.409	11.460
2050.	11.432	12.620	2550.	10.378	11.415
2060.	11.394	12.598	2560.	10.346	11.367
2070.	11.358	12.575	2570.	10.310	11.318
2080.	11.322	12.554	2580.	10.273	11.266
2090.	11.288	12.533	2590.	10.232	11.212
2100.	11.255	12.513	2600.	10.190	11.157
2110.	11.223	12.493	2610.	10.144	11.099
2120.	11.192	12.474	2620.	10.096	11.040
2130.	11.162	12.455	2630.	10.046	10.978
2140.	11.133	12.436	2640.	9.993	10.915
2150.	11.105	12.418	2650.	9.937	10.850
2160.	11.078	12.400	2660.	9.879	10.783
2170.	11.053	12.382	2670.	9.819	10.714
2180.	11.028	12.364	2680.	9.756	10.645
2190.	11.004	12.346	2690.	9.692	10.574
2200.	10.981	12.329	2700.	9.625	10.503
2210.	10.960	12.311	2710.	9.557	10.430
2220.	10.939	12.294	2720.	9.488	10.358
2230.	10.919	12.276	2730.	9.417	10.285
2240.	10.900	12.259	2740.	9.346	10.212
2250.	10.881	12.241	2750.	9.274	10.139
2260.	10.864	12.223	2760.	9.202	10.068
2270.	10.847	12.205	2770.	9.130	9.997
2280.	10.831	12.187	2780.	9.057	9.927
2290.	10.816	12.168	2790.	8.985	9.858
2300.	10.802	12.149	2800.	8.914	9.791
2310.	10.787	12.130	2810.	8.843	9.726
2320.	10.774	12.110	2820.	8.774	9.663
2330.	10.761	12.090	2830.	8.705	9.601
2340.	10.748	12.069	2840.	8.638	9.542
2350.	10.736	12.048	2850.	8.572	9.484
2360.	10.723	12.026	2860.	8.507	9.429
2370.	10.711	12.004	2870.	8.444	9.376
2380.	10.699	11.981	2880.	8.382	9.326
2390.	10.687	11.957	2890.	8.322	9.277
2400.	10.674	11.932	2900.	8.264	9.231
2410.	10.661	11.906	2910.	8.207	9.187
2420.	10.648	11.880	2920.	8.151	9.144
2430.	10.634	11.852	2930.	8.097	9.104
2440.	10.620	11.823	2940.	8.044	9.066
2450.	10.605	11.794	2950.	7.993	9.029
2460.	10.589	11.762	2960.	7.943	8.994
2470.	10.571	11.730	2970.	7.895	8.961
2480.	10.553	11.696	2980.	7.848	8.929
2490.	10.533	11.661	2990.	7.802	8.899

Table 3 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
3000.	7.758	8.870	3500.	6.633	7.745
3010.	7.714	8.842	3510.	6.610	7.707
3020.	7.672	8.816	3520.	6.585	7.667
3030.	7.631	8.790	3530.	6.559	7.625
3040.	7.591	8.765	3540.	6.530	7.581
3050.	7.553	8.742	3550.	6.500	7.536
3060.	7.515	8.719	3560.	6.467	7.488
3070.	7.479	8.697	3570.	6.432	7.439
3080.	7.444	8.675	3580.	6.394	7.387
3090.	7.409	8.654	3590.	6.354	7.334
3100.	7.376	8.634	3600.	6.311	7.278
3110.	7.344	8.614	3610.	6.266	7.220
3120.	7.313	8.595	3620.	6.218	7.161
3130.	7.283	8.576	3630.	6.167	7.099
3140.	7.254	8.557	3640.	6.114	7.036
3150.	7.227	8.539	3650.	6.058	6.971
3160.	7.200	8.521	3660.	6.000	6.904
3170.	7.174	8.503	3670.	5.940	6.836
3180.	7.149	8.485	3680.	5.877	6.766
3190.	7.125	8.468	3690.	5.813	6.695
3200.	7.103	8.450	3700.	5.746	6.624
3210.	7.081	8.433	3710.	5.678	6.551
3220.	7.060	8.415	3720.	5.609	6.479
3230.	7.040	8.398	3730.	5.539	6.406
3240.	7.021	8.380	3740.	5.467	6.333
3250.	7.003	8.362	3750.	5.395	6.261
3260.	6.985	8.344	3760.	5.323	6.189
3270.	6.968	8.326	3770.	5.251	6.118
3280.	6.953	8.308	3780.	5.178	6.048
3290.	6.937	8.289	3790.	5.107	5.980
3300.	6.923	8.270	3800.	5.035	5.913
3310.	6.909	8.251	3810.	4.965	5.847
3320.	6.895	8.231	3820.	4.895	5.784
3330.	6.882	8.211	3830.	4.826	5.722
3340.	6.869	8.190	3840.	4.759	5.663
3350.	6.857	8.169	3850.	4.693	5.606
3360.	6.845	8.147	3860.	4.629	5.550
3370.	6.832	8.125	3870.	4.565	5.498
3380.	6.820	8.102	3880.	4.504	5.447
3390.	6.808	8.078	3890.	4.444	5.398
3400.	6.795	8.053	3900.	4.385	5.352
3410.	6.783	8.028	3910.	4.328	5.308
3420.	6.769	8.001	3920.	4.272	5.266
3430.	6.756	7.973	3930.	4.218	5.225
3440.	6.741	7.945	3940.	4.166	5.187
3450.	6.726	7.915	3950.	4.114	5.150
3460.	6.710	7.884	3960.	4.065	5.116
3470.	6.693	7.851	3970.	4.016	5.082
3480.	6.674	7.817	3980.	3.969	5.051
3490.	6.654	7.782	3990.	3.923	5.020

Table 3 continued

x/m	bed lev/m	stage/m
4000.	3.879	4.991
4010.	3.835	4.963
4020.	3.793	4.937
4030.	3.752	4.911
4040.	3.713	4.887
4050.	3.674	4.863
4060.	3.637	4.840
4070.	3.600	4.818
4080.	3.565	4.796
4090.	3.531	4.775
4100.	3.497	4.755
4110.	3.465	4.735
4120.	3.434	4.716
4130.	3.404	4.697
4140.	3.376	4.678
4150.	3.348	4.660
4160.	3.321	4.642
4170.	3.295	4.624
4180.	3.270	4.606
4190.	3.247	4.589
4200.	3.224	4.571
4210.	3.202	4.554
4220.	3.181	4.536
4230.	3.161	4.519
4240.	3.142	4.501
4250.	3.124	4.483
4260.	3.106	4.465
4270.	3.090	4.447
4280.	3.074	4.429
4290.	3.059	4.410
4300.	3.044	4.392
4310.	3.030	4.372
4320.	3.016	4.353
4330.	3.003	4.332
4340.	2.991	4.312
4350.	2.978	4.290
4360.	2.966	4.269
4370.	2.954	4.246
4380.	2.941	4.223
4390.	2.929	4.199
4400.	2.917	4.174
4410.	2.904	4.149
4420.	2.891	4.122
4430.	2.877	4.095
4440.	2.862	4.066
4450.	2.847	4.036
4460.	2.831	4.005
4470.	2.814	3.972
4480.	2.795	3.939
4490.	2.775	3.903

x/m	bed lev/m	stage/m
4500.	2.754	3.867
4510.	2.731	3.828
4520.	2.707	3.788
4530.	2.680	3.746
4540.	2.652	3.703
4550.	2.621	3.657
4560.	2.588	3.609
4570.	2.553	3.560
4580.	2.515	3.508
4590.	2.475	3.455
4600.	2.432	3.399
4610.	2.387	3.342
4620.	2.339	3.282
4630.	2.288	3.221
4640.	2.235	3.157
4650.	2.180	3.092
4660.	2.121	3.025
4670.	2.061	2.957
4680.	1.999	2.887
4690.	1.934	2.817
4700.	1.868	2.745
4710.	1.800	2.673
4720.	1.730	2.600
4730.	1.660	2.527
4740.	1.589	2.454
4750.	1.517	2.382
4760.	1.444	2.310
4770.	1.372	2.239
4780.	1.300	2.169
4790.	1.228	2.101
4800.	1.156	2.034
4810.	1.086	1.968
4820.	1.016	1.905
4830.	0.948	1.843
4840.	0.880	1.784
4850.	0.814	1.727
4860.	0.750	1.672
4870.	0.687	1.619
4880.	0.625	1.568
4890.	0.565	1.520
4900.	0.506	1.473
4910.	0.449	1.429
4920.	0.394	1.387
4930.	0.339	1.347
4940.	0.287	1.308
4950.	0.236	1.272
4960.	0.186	1.237
4970.	0.137	1.204
4980.	0.090	1.172
4990.	0.045	1.141
5000.	0.000	1.112

Table 3 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	37.293	37.874	2550.	16.009	16.686
50.	36.737	37.321	2600.	15.663	16.341
100.	36.191	36.779	2650.	15.318	15.998
150.	35.655	36.246	2700.	14.975	15.655
200.	35.128	35.723	2750.	14.633	15.314
250.	34.611	35.209	2800.	14.292	14.974
300.	34.103	34.704	2850.	13.953	14.635
350.	33.602	34.206	2900.	13.614	14.297
400.	33.109	33.717	2950.	13.276	13.959
450.	32.624	33.234	3000.	12.939	13.623
500.	32.145	32.758	3050.	12.603	13.288
550.	31.674	32.289	3100.	12.268	12.954
600.	31.209	31.827	3150.	11.934	12.620
650.	30.749	31.370	3200.	11.601	12.288
700.	30.296	30.919	3250.	11.269	11.956
750.	29.848	30.474	3300.	10.937	11.625
800.	29.405	30.033	3350.	10.607	11.295
850.	28.968	29.598	3400.	10.277	10.965
900.	28.535	29.168	3450.	9.947	10.636
950.	28.107	28.742	3500.	9.619	10.308
1000.	27.683	28.320	3550.	9.291	9.981
1050.	27.264	27.903	3600.	8.964	9.654
1100.	26.848	27.489	3650.	8.637	9.328
1150.	26.437	27.080	3700.	8.311	9.002
1200.	26.029	26.674	3750.	7.986	8.677
1250.	25.625	26.271	3800.	7.661	8.353
1300.	25.224	25.872	3850.	7.337	8.029
1350.	24.827	25.476	3900.	7.013	7.706
1400.	24.432	25.084	3950.	6.690	7.383
1450.	24.041	24.694	4000.	6.367	7.061
1500.	23.653	24.307	4050.	6.045	6.739
1550.	23.267	23.923	4100.	5.723	6.417
1600.	22.884	23.542	4150.	5.402	6.097
1650.	22.504	23.163	4200.	5.081	5.776
1700.	22.126	22.786	4250.	4.761	5.456
1750.	21.751	22.412	4300.	4.441	5.136
1800.	21.378	22.040	4350.	4.122	4.817
1850.	21.007	21.671	4400.	3.803	4.498
1900.	20.638	21.303	4450.	3.484	4.180
1950.	20.272	20.938	4500.	3.166	3.862
2000.	19.907	20.574	4550.	2.848	3.544
2050.	19.544	20.213	4600.	2.530	3.227
2100.	19.184	19.853	4650.	2.213	2.910
2150.	18.824	19.495	4700.	1.896	2.593
2200.	18.467	19.139	4750.	1.579	2.276
2250.	18.111	18.784	4800.	1.263	1.960
2300.	17.757	18.431	4850.	0.946	1.644
2350.	17.405	18.079	4900.	0.631	1.329
2400.	17.053	17.729	4950.	0.315	1.013
2450.	16.704	17.380	5000.	0.000	0.698
2500.	16.355	17.032			

Table 4: Bed level and stage data for UR5

x/m	bed lev/m	stage/m
0.	4.043	5.001
10.	4.023	4.974
20.	4.003	4.947
30.	3.982	4.919
40.	3.960	4.890
50.	3.937	4.861
60.	3.914	4.830
70.	3.890	4.799
80.	3.866	4.768
90.	3.840	4.735
100.	3.814	4.702
110.	3.787	4.668
120.	3.759	4.633
130.	3.730	4.597
140.	3.701	4.560
150.	3.670	4.522
160.	3.638	4.483
170.	3.606	4.443
180.	3.572	4.402
190.	3.537	4.360
200.	3.501	4.316
210.	3.463	4.271
220.	3.424	4.225
230.	3.384	4.178
240.	3.343	4.129
250.	3.300	4.079
260.	3.255	4.027
270.	3.209	3.973
280.	3.162	3.918
290.	3.112	3.861
300.	3.061	3.803
310.	3.008	3.742
320.	2.953	3.680
330.	2.896	3.616
340.	2.837	3.550
350.	2.776	3.482
360.	2.713	3.412
370.	2.648	3.340
380.	2.581	3.267
390.	2.512	3.192
400.	2.441	3.116
410.	2.368	3.038
420.	2.293	2.959
430.	2.217	2.878
440.	2.140	2.797
450.	2.061	2.714
460.	1.981	2.630
470.	1.899	2.546
480.	1.816	2.460
490.	1.733	2.374
500.	1.648	2.287
510.	1.563	2.199
520.	1.476	2.111
530.	1.390	2.022
540.	1.302	1.933
550.	1.214	1.844
560.	1.125	1.754
570.	1.036	1.663
580.	0.947	1.573
590.	0.857	1.482
600.	0.766	1.391
610.	0.683	1.674
620.	0.621	1.682
630.	0.577	1.679
640.	0.544	1.672
650.	0.518	1.663
660.	0.496	1.653
670.	0.477	1.643
680.	0.460	1.632
690.	0.443	1.621
700.	0.427	1.610
710.	0.412	1.599
720.	0.397	1.588
730.	0.381	1.577
740.	0.366	1.566
750.	0.351	1.556
760.	0.336	1.546
770.	0.322	1.535
780.	0.307	1.525
790.	0.292	1.515
800.	0.278	1.505
810.	0.263	1.496
820.	0.249	1.486
830.	0.234	1.477
840.	0.220	1.467
850.	0.206	1.458
860.	0.192	1.449
870.	0.178	1.440
880.	0.164	1.431
890.	0.150	1.423
900.	0.136	1.414
910.	0.122	1.406
920.	0.108	1.397
930.	0.094	1.389
940.	0.081	1.381
950.	0.067	1.373
960.	0.054	1.365
970.	0.040	1.357
980.	0.027	1.350
990.	0.013	1.342
1000.	0.000	1.335

Table 5: Bed level and stage data for UR9

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	12.920	14.045	500.	12.231	13.356
10.	12.887	14.028	510.	12.221	13.331
20.	12.855	14.011	520.	12.210	13.304
30.	12.824	13.995	530.	12.198	13.276
40.	12.793	13.980	540.	12.184	13.247
50.	12.764	13.966	550.	12.168	13.216
60.	12.735	13.952	560.	12.151	13.184
70.	12.707	13.939	570.	12.131	13.150
80.	12.680	13.926	580.	12.110	13.114
90.	12.654	13.913	590.	12.087	13.078
100.	12.629	13.901	600.	12.061	13.039
110.	12.605	13.890	610.	12.034	12.999
120.	12.582	13.878	620.	12.004	12.958
130.	12.560	13.867	630.	11.972	12.915
140.	12.538	13.856	640.	11.938	12.870
150.	12.518	13.845	650.	11.902	12.825
160.	12.499	13.835	660.	11.864	12.778
170.	12.480	13.824	670.	11.824	12.730
180.	12.463	13.814	680.	11.782	12.681
190.	12.446	13.803	690.	11.738	12.631
200.	12.430	13.793	700.	11.693	12.580
210.	12.416	13.783	710.	11.646	12.529
220.	12.402	13.773	720.	11.598	12.477
230.	12.389	13.762	730.	11.549	12.426
240.	12.378	13.752	740.	11.499	12.374
250.	12.367	13.742	750.	11.448	12.323
260.	12.357	13.731	760.	11.397	12.272
270.	12.347	13.720	770.	11.345	12.222
280.	12.339	13.709	780.	11.294	12.173
290.	12.331	13.698	790.	11.242	12.125
300.	12.324	13.687	800.	11.191	12.078
310.	12.318	13.676	810.	11.140	12.033
320.	12.312	13.664	820.	11.090	11.989
330.	12.307	13.652	830.	11.040	11.946
340.	12.303	13.639	840.	10.991	11.905
350.	12.299	13.626	850.	10.943	11.866
360.	12.295	13.613	860.	10.896	11.828
370.	12.292	13.599	870.	10.849	11.792
380.	12.289	13.585	880.	10.804	11.758
390.	12.285	13.570	890.	10.760	11.725
400.	12.282	13.554	900.	10.716	11.694
410.	12.279	13.538	910.	10.674	11.665
420.	12.276	13.522	920.	10.633	11.637
430.	12.273	13.504	930.	10.592	11.611
440.	12.269	13.486	940.	10.553	11.586
450.	12.264	13.467	950.	10.514	11.562
460.	12.259	13.447	960.	10.477	11.540
470.	12.254	13.426	970.	10.440	11.518
480.	12.247	13.404	980.	10.405	11.498
490.	12.240	13.380	990.	10.370	11.479

Table 6: Bed level and stage data for UT2

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
1000.	10.336	11.461	1500.	9.647	10.772
1010.	10.303	11.444	1510.	9.637	10.747
1020.	10.271	11.427	1520.	9.626	10.720
1030.	10.240	11.411	1530.	9.614	10.692
1040.	10.209	11.396	1540.	9.600	10.663
1050.	10.180	11.382	1550.	9.584	10.632
1060.	10.151	11.368	1560.	9.567	10.600
1070.	10.123	11.355	1570.	9.547	10.566
1080.	10.096	11.342	1580.	9.526	10.530
1090.	10.070	11.329	1590.	9.503	10.494
1100.	10.045	11.317	1600.	9.477	10.455
1110.	10.021	11.306	1610.	9.450	10.415
1120.	9.998	11.294	1620.	9.420	10.374
1130.	9.976	11.283	1630.	9.388	10.331
1140.	9.954	11.272	1640.	9.354	10.286
1150.	9.934	11.261	1650.	9.318	10.241
1160.	9.915	11.251	1660.	9.280	10.194
1170.	9.896	11.240	1670.	9.240	10.146
1180.	9.879	11.230	1680.	9.198	10.097
1190.	9.862	11.219	1690.	9.154	10.047
1200.	9.846	11.209	1700.	9.109	9.996
1210.	9.832	11.199	1710.	9.062	9.945
1220.	9.818	11.189	1720.	9.014	9.893
1230.	9.805	11.178	1730.	8.965	9.842
1240.	9.794	11.168	1740.	8.915	9.790
1250.	9.783	11.158	1750.	8.864	9.739
1260.	9.773	11.147	1760.	8.813	9.688
1270.	9.763	11.136	1770.	8.761	9.638
1280.	9.755	11.125	1780.	8.710	9.589
1290.	9.747	11.114	1790.	8.658	9.541
1300.	9.740	11.103	1800.	8.607	9.494
1310.	9.734	11.092	1810.	8.556	9.449
1320.	9.728	11.080	1820.	8.506	9.405
1330.	9.723	11.068	1830.	8.456	9.362
1340.	9.719	11.055	1840.	8.407	9.321
1350.	9.715	11.042	1850.	8.359	9.282
1360.	9.711	11.029	1860.	8.312	9.244
1370.	9.708	11.015	1870.	8.265	9.208
1380.	9.705	11.001	1880.	8.220	9.174
1390.	9.701	10.986	1890.	8.176	9.141
1400.	9.698	10.970	1900.	8.132	9.110
1410.	9.695	10.954	1910.	8.090	9.081
1420.	9.692	10.938	1920.	8.049	9.053
1430.	9.689	10.920	1930.	8.008	9.027
1440.	9.685	10.902	1940.	7.969	9.002
1450.	9.680	10.883	1950.	7.930	8.978
1460.	9.675	10.863	1960.	7.893	8.956
1470.	9.670	10.842	1970.	7.856	8.934
1480.	9.663	10.820	1980.	7.821	8.914
1490.	9.656	10.796	1990.	7.786	8.895

Table 6 continued

x/m	**bed lev/m**	**stage/m**	**x/m**	**bed lev/m**	**stage/m**
2000.	7.752	8.877	2500.	7.063	8.188
2010.	7.719	8.860	2510.	7.053	8.163
2020.	7.687	8.843	2520.	7.042	8.136
2030.	7.656	8.827	2530.	7.030	8.108
2040.	7.625	8.812	2540.	7.016	8.079
2050.	7.596	8.798	2550.	7.000	8.048
2060.	7.567	8.784	2560.	6.983	8.016
2070.	7.539	8.771	2570.	6.963	7.982
2080.	7.512	8.758	2580.	6.942	7.946
2090.	7.486	8.745	2590.	6.919	7.910
2100.	7.461	8.733	2600.	6.893	7.871
2110.	7.437	8.722	2610.	6.866	7.831
2120.	7.414	8.710	2620.	6.836	7.790
2130.	7.392	8.699	2630.	6.804	7.747
2140.	7.370	8.688	2640.	6.770	7.703
2150.	7.350	8.677	2650.	6.734	7.657
2160.	7.331	8.667	2660.	6.696	7.610
2170.	7.312	8.656	2670.	6.656	7.562
2180.	7.295	8.646	2680.	6.614	7.513
2190.	7.278	8.635	2690.	6.570	7.463
2200.	7.262	8.625	2700.	6.525	7.412
2210.	7.248	8.615	2710.	6.478	7.361
2220.	7.234	8.605	2720.	6.430	7.309
2230.	7.221	8.594	2730.	6.381	7.258
2240.	7.210	8.584	2740.	6.331	7.206
2250.	7.199	8.574	2750.	6.280	7.155
2260.	7.189	8.563	2760.	6.229	7.104
2270.	7.179	8.552	2770.	6.177	7.054
2280.	7.171	8.541	2780.	6.126	7.005
2290.	7.163	8.530	2790.	6.074	6.957
2300.	7.156	8.519	2800.	6.023	6.910
2310.	7.150	8.508	2810.	5.972	6.865
2320.	7.144	8.496	2820.	5.922	6.821
2330.	7.139	8.484	2830.	5.872	6.778
2340.	7.135	8.471	2840.	5.823	6.737
2350.	7.131	8.458	2850.	5.775	6.698
2360.	7.127	8.445	2860.	5.728	6.660
2370.	7.124	8.431	2870.	5.681	6.624
2380.	7.121	8.417	2880.	5.636	6.590
2390.	7.117	8.402	2890.	5.592	6.557
2400.	7.114	8.386	2900.	5.548	6.526
2410.	7.111	8.370	2910.	5.506	6.497
2420.	7.108	8.354	2920.	5.465	6.469
2430.	7.105	8.336	2930.	5.424	6.443
2440.	7.101	8.318	2940.	5.385	6.418
2450.	7.096	8.299	2950.	5.346	6.394
2460.	7.091	8.279	2960.	5.309	6.372
2470.	7.086	8.258	2970.	5.272	6.350
2480.	7.079	8.236	2980.	5.237	6.330
2490.	7.072	8.212	2990.	5.202	6.311

Table 6 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
3000.	5.168	6.293	3500.	4.479	5.604
3010.	5.135	6.276	3510.	4.469	5.579
3020.	5.103	6.259	3520.	4.458	5.552
3030.	5.072	6.243	3530.	4.446	5.524
3040.	5.041	6.228	3540.	4.432	5.495
3050.	5.012	6.214	3550.	4.416	5.464
3060.	4.983	6.200	3560.	4.399	5.432
3070.	4.955	6.187	3570.	4.379	5.398
3080.	4.928	6.174	3580.	4.358	5.363
3090.	4.902	6.161	3590.	4.335	5.326
3100.	4.877	6.149	3600.	4.309	5.287
3110.	4.853	6.138	3610.	4.282	5.247
3120.	4.830	6.126	3620.	4.252	5.206
3130.	4.808	6.115	3630.	4.220	5.163
3140.	4.786	6.104	3640.	4.186	5.119
3150.	4.766	6.093	3650.	4.150	5.073
3160.	4.747	6.083	3660.	4.112	5.026
3170.	4.728	6.072	3670.	4.072	4.978
3180.	4.711	6.062	3680.	4.030	4.929
3190.	4.694	6.051	3690.	3.986	4.879
3200.	4.678	6.041	3700.	3.941	4.828
3210.	4.664	6.031	3710.	3.894	4.777
3220.	4.650	6.021	3720.	3.846	4.725
3230.	4.637	6.010	3730.	3.797	4.674
3240.	4.626	6.000	3740.	3.747	4.622
3250.	4.615	5.990	3750.	3.696	4.571
3260.	4.605	5.979	3760.	3.645	4.520
3270.	4.595	5.968	3770.	3.593	4.470
3280.	4.587	5.957	3780.	3.542	4.421
3290.	4.579	5.946	3790.	3.490	4.373
3300.	4.572	5.935	3800.	3.439	4.326
3310.	4.566	5.924	3810.	3.388	4.281
3320.	4.560	5.912	3820.	3.338	4.237
3330.	4.555	5.900	3830.	3.288	4.194
3340.	4.551	5.887	3840.	3.239	4.153
3350.	4.547	5.874	3850.	3.191	4.114
3360.	4.543	5.861	3860.	3.144	4.076
3370.	4.540	5.847	3870.	3.097	4.040
3380.	4.537	5.833	3880.	3.052	4.006
3390.	4.533	5.818	3890.	3.008	3.973
3400.	4.530	5.802	3900.	2.964	3.942
3410.	4.527	5.786	3910.	2.922	3.913
3420.	4.524	5.770	3920.	2.881	3.885
3430.	4.521	5.752	3930.	2.840	3.859
3440.	4.517	5.734	3940.	2.801	3.834
3450.	4.512	5.715	3950.	2.762	3.810
3460.	4.507	5.695	3960.	2.725	3.788
3470.	4.502	5.674	3970.	2.688	3.766
3480.	4.495	5.652	3980.	2.653	3.746
3490.	4.488	5.628	3990.	2.618	3.727

Table 6 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
4000.	2.584	3.709	4500.	1.895	3.020
4010.	2.551	3.692	4510.	1.885	2.995
4020.	2.519	3.675	4520.	1.874	2.968
4030.	2.488	3.659	4530.	1.862	2.940
4040.	2.457	3.644	4540.	1.848	2.911
4050.	2.428	3.630	4550.	1.832	2.880
4060.	2.399	3.616	4560.	1.815	2.848
4070.	2.371	3.603	4570.	1.795	2.814
4080.	2.344	3.590	4580.	1.774	2.779
4090.	2.318	3.577	4590.	1.751	2.742
4100.	2.293	3.565	4600.	1.725	2.703
4110.	2.269	3.554	4610.	1.698	2.663
4120.	2.246	3.542	4620.	1.668	2.622
4130.	2.224	3.531	4630.	1.636	2.579
4140.	2.202	3.520	4640.	1.602	2.535
4150.	2.182	3.509	4650.	1.566	2.489
4160.	2.163	3.499	4660.	1.528	2.442
4170.	2.144	3.488	4670.	1.488	2.394
4180.	2.127	3.478	4680.	1.446	2.345
4190.	2.110	3.467	4690.	1.402	2.295
4200.	2.094	3.457	4700.	1.357	2.244
4210.	2.080	3.447	4710.	1.310	2.193
4220.	2.066	3.437	4720.	1.262	2.141
4230.	2.053	3.426	4730.	1.213	2.090
4240.	2.042	3.416	4740.	1.163	2.038
4250.	2.031	3.406	4750.	1.112	1.987
4260.	2.021	3.395	4760.	1.061	1.936
4270.	2.011	3.384	4770.	1.009	1.886
4280.	2.003	3.373	4780.	0.958	1.837
4290.	1.995	3.362	4790.	0.906	1.789
4300.	1.988	3.351	4800.	0.855	1.742
4310.	1.982	3.340	4810.	0.804	1.697
4320.	1.977	3.328	4820.	0.754	1.653
4330.	1.971	3.316	4830.	0.704	1.610
4340.	1.967	3.303	4840.	0.655	1.569
4350.	1.963	3.290	4850.	0.607	1.530
4360.	1.959	3.277	4860.	0.560	1.492
4370.	1.956	3.263	4870.	0.513	1.456
4380.	1.953	3.249	4880.	0.468	1.422
4390.	1.949	3.234	4890.	0.424	1.389
4400.	1.946	3.218	4900.	0.380	1.358
4410.	1.943	3.202	4910.	0.338	1.329
4420.	1.940	3.186	4920.	0.297	1.301
4430.	1.937	3.168	4930.	0.256	1.275
4440.	1.933	3.150	4940.	0.217	1.250
4450.	1.928	3.131	4950.	0.178	1.226
4460.	1.923	3.111	4960.	0.141	1.204
4470.	1.918	3.090	4970.	0.104	1.182
4480.	1.911	3.068	4980.	0.069	1.162
4490.	1.904	3.044	4990.	0.034	1.143
			5000.	0.000	1.125

Table 6 continued

x/m	bed lev/m	stage/m
0.	3.719	4.653
10.	3.702	4.629
20.	3.684	4.605
30.	3.666	4.580
40.	3.647	4.555
50.	3.628	4.528
60.	3.608	4.502
70.	3.587	4.474
80.	3.566	4.446
90.	3.544	4.417
100.	3.521	4.387
110.	3.497	4.356
120.	3.473	4.325
130.	3.447	4.293
140.	3.421	4.259
150.	3.394	4.225
160.	3.366	4.190
170.	3.337	4.154
180.	3.306	4.116
190.	3.275	4.078
200.	3.243	4.038
210.	3.209	3.997
220.	3.174	3.955
230.	3.138	3.912
240.	3.101	3.867
250.	3.062	3.821
260.	3.021	3.774
270.	2.979	3.725
280.	2.936	3.674
290.	2.891	3.621
300.	2.844	3.567
310.	2.795	3.511
320.	2.745	3.454
330.	2.692	3.394
340.	2.637	3.332
350.	2.581	3.269
360.	2.522	3.204
370.	2.462	3.138
380.	2.399	3.069
390.	2.335	2.999
400.	2.269	2.928
410.	2.201	2.855
420.	2.132	2.781
430.	2.060	2.705
440.	1.988	2.628
450.	1.914	2.551
460.	1.838	2.472
470.	1.762	2.393
480.	1.684	2.312
490.	1.606	2.231
500.	1.526	2.149
510.	1.446	2.067
520.	1.365	1.984
530.	1.283	1.900
540.	1.200	1.816
550.	1.117	1.732
560.	1.034	1.647
570.	0.950	1.562
580.	0.866	1.477
590.	0.781	1.391
600.	0.696	1.305
610.	0.617	1.581
620.	0.558	1.590
630.	0.514	1.590
640.	0.482	1.586
650.	0.455	1.580
660.	0.433	1.573
670.	0.413	1.566
680.	0.396	1.558
690.	0.379	1.551
700.	0.364	1.543
710.	0.349	1.535
720.	0.334	1.528
730.	0.320	1.520
740.	0.307	1.513
750.	0.293	1.505
760.	0.280	1.498
770.	0.268	1.491
780.	0.255	1.483
790.	0.243	1.476
800.	0.230	1.469
810.	0.218	1.463
820.	0.206	1.456
830.	0.194	1.449
840.	0.182	1.443
850.	0.170	1.436
860.	0.159	1.430
870.	0.147	1.423
880.	0.135	1.417
890.	0.124	1.411
900.	0.112	1.405
910.	0.101	1.399
920.	0.090	1.393
930.	0.078	1.388
940.	0.067	1.382
950.	0.056	1.376
960.	0.045	1.371
970.	0.033	1.366
980.	0.022	1.360
990.	0.011	1.355
1000.	0.000	1.350

Table 7: Bed level and stage data for UT4

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	24.920	25.921	500.	22.619	23.634
10.	24.874	25.875	510.	22.572	23.589
20.	24.828	25.829	520.	22.526	23.543
30.	24.782	25.783	530.	22.480	23.498
40.	24.736	25.737	540.	22.434	23.453
50.	24.691	25.691	550.	22.387	23.408
60.	24.645	25.645	560.	22.341	23.363
70.	24.599	25.599	570.	22.295	23.318
80.	24.553	25.554	580.	22.248	23.273
90.	24.507	25.508	590.	22.202	23.228
100.	24.461	25.462	600.	22.155	23.183
110.	24.415	25.416	610.	22.109	23.138
120.	24.369	25.370	620.	22.063	23.093
130.	24.323	25.324	630.	22.016	23.048
140.	24.277	25.278	640.	21.970	23.003
150.	24.231	25.232	650.	21.923	22.958
160.	24.185	25.187	660.	21.876	22.914
170.	24.139	25.141	670.	21.830	22.869
180.	24.093	25.095	680.	21.783	22.825
190.	24.047	25.049	690.	21.737	22.780
200.	24.001	25.003	700.	21.690	22.736
210.	23.955	24.957	710.	21.643	22.691
220.	23.909	24.912	720.	21.596	22.647
230.	23.863	24.866	730.	21.550	22.602
240.	23.817	24.820	740.	21.503	22.558
250.	23.771	24.774	750.	21.456	22.514
260.	23.725	24.728	760.	21.409	22.470
270.	23.679	24.683	770.	21.362	22.426
280.	23.633	24.637	780.	21.315	22.382
290.	23.587	24.591	790.	21.268	22.338
300.	23.541	24.545	800.	21.221	22.294
310.	23.495	24.500	810.	21.174	22.250
320.	23.449	24.454	820.	21.126	22.206
330.	23.403	24.408	830.	21.079	22.162
340.	23.357	24.363	840.	21.032	22.118
350.	23.311	24.317	850.	20.984	22.075
360.	23.265	24.271	860.	20.937	22.031
370.	23.219	24.226	870.	20.889	21.987
380.	23.173	24.180	880.	20.841	21.944
390.	23.127	24.134	890.	20.793	21.900
400.	23.081	24.089	900.	20.746	21.857
410.	23.034	24.043	910.	20.698	21.813
420.	22.988	23.998	920.	20.649	21.769
430.	22.942	23.952	930.	20.601	21.726
440.	22.896	23.907	940.	20.553	21.682
450.	22.850	23.861	950.	20.504	21.639
460.	22.804	23.816	960.	20.456	21.595
470.	22.757	23.770	970.	20.407	21.551
480.	22.711	23.725	980.	20.358	21.508
490.	22.665	23.679	990.	20.309	21.464

Table 8: Bed level and stage data for VR1

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
1000.	20.260	21.420	1500.	17.349	18.815
1010.	20.210	21.376	1510.	17.278	18.748
1020.	20.161	21.332	1520.	17.208	18.681
1030.	20.111	21.288	1530.	17.136	18.613
1040.	20.061	21.244	1540.	17.065	18.545
1050.	20.011	21.200	1550.	16.993	18.476
1060.	19.960	21.155	1560.	16.922	18.407
1070.	19.910	21.111	1570.	16.850	18.338
1080.	19.859	21.066	1580.	16.778	18.268
1090.	19.808	21.021	1590.	16.706	18.198
1100.	19.756	20.976	1600.	16.634	18.128
1110.	19.705	20.931	1610.	16.562	18.058
1120.	19.653	20.885	1620.	16.490	17.988
1130.	19.601	20.840	1630.	16.419	17.917
1140.	19.548	20.794	1640.	16.348	17.847
1150.	19.495	20.748	1650.	16.277	17.777
1160.	19.442	20.701	1660.	16.206	17.707
1170.	19.388	20.654	1670.	16.136	17.637
1180.	19.334	20.607	1680.	16.067	17.567
1190.	19.280	20.559	1690.	15.998	17.498
1200.	19.225	20.511	1700.	15.930	17.429
1210.	19.170	20.463	1710.	15.862	17.360
1220.	19.114	20.414	1720.	15.795	17.292
1230.	19.058	20.365	1730.	15.729	17.224
1240.	19.002	20.316	1740.	15.664	17.157
1250.	18.945	20.266	1750.	15.599	17.091
1260.	18.888	20.215	1760.	15.535	17.025
1270.	18.830	20.164	1770.	15.472	16.960
1280.	18.771	20.112	1780.	15.410	16.895
1290.	18.713	20.060	1790.	15.349	16.831
1300.	18.653	20.008	1800.	15.289	16.768
1310.	18.593	19.954	1810.	15.230	16.706
1320.	18.533	19.900	1820.	15.172	16.644
1330.	18.472	19.846	1830.	15.114	16.583
1340.	18.410	19.791	1840.	15.058	16.523
1350.	18.348	19.735	1850.	15.003	16.463
1360.	18.285	19.678	1860.	14.948	16.405
1370.	18.222	19.621	1870.	14.895	16.347
1380.	18.158	19.563	1880.	14.842	16.289
1390.	18.094	19.505	1890.	14.790	16.233
1400.	18.029	19.445	1900.	14.739	16.177
1410.	17.963	19.386	1910.	14.689	16.122
1420.	17.897	19.325	1920.	14.640	16.067
1430.	17.830	19.263	1930.	14.592	16.014
1440.	17.763	19.201	1940.	14.544	15.960
1450.	17.695	19.139	1950.	14.497	15.908
1460.	17.627	19.075	1960.	14.451	15.856
1470.	17.558	19.011	1970.	14.405	15.805
1480.	17.489	18.946	1980.	14.360	15.754
1490.	17.419	18.881	1990.	14.316	15.704

Table 8 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
2000.	14.273	15.654	2500.	12.279	13.448
2010.	14.230	15.605	2510.	12.236	13.405
2020.	14.187	15.556	2520.	12.193	13.362
2030.	14.145	15.508	2530.	12.149	13.319
2040.	14.103	15.460	2540.	12.105	13.276
2050.	14.062	15.413	2550.	12.061	13.232
2060.	14.021	15.366	2560.	12.016	13.189
2070.	13.981	15.319	2570.	11.971	13.146
2080.	13.941	15.273	2580.	11.926	13.102
2090.	13.901	15.227	2590.	11.881	13.059
2100.	13.862	15.181	2600.	11.835	13.015
2110.	13.823	15.136	2610.	11.789	12.972
2120.	13.784	15.091	2620.	11.743	12.928
2130.	13.745	15.046	2630.	11.697	12.884
2140.	13.706	15.001	2640.	11.650	12.840
2150.	13.668	14.957	2650.	11.603	12.797
2160.	13.630	14.913	2660.	11.556	12.753
2170.	13.592	14.869	2670.	11.508	12.709
2180.	13.553	14.825	2680.	11.461	12.664
2190.	13.515	14.781	2690.	11.413	12.620
2200.	13.477	14.737	2700.	11.364	12.576
2210.	13.439	14.694	2710.	11.315	12.531
2220.	13.401	14.651	2720.	11.266	12.486
2230.	13.363	14.607	2730.	11.217	12.442
2240.	13.325	14.564	2740.	11.167	12.397
2250.	13.287	14.521	2750.	11.117	12.351
2260.	13.249	14.478	2760.	11.067	12.306
2270.	13.210	14.435	2770.	11.016	12.260
2280.	13.172	14.392	2780.	10.965	12.215
2290.	13.133	14.349	2790.	10.914	12.169
2300.	13.095	14.306	2800.	10.862	12.122
2310.	13.056	14.263	2810.	10.810	12.076
2320.	13.017	14.221	2820.	10.758	12.029
2330.	12.978	14.178	2830.	10.705	11.982
2340.	12.938	14.135	2840.	10.651	11.934
2350.	12.899	14.092	2850.	10.598	11.887
2360.	12.859	14.049	2860.	10.544	11.838
2370.	12.819	14.007	2870.	10.489	11.790
2380.	12.779	13.964	2880.	10.434	11.741
2390.	12.739	13.921	2890.	10.378	11.692
2400.	12.698	13.878	2900.	10.322	11.642
2410.	12.657	13.835	2910.	10.266	11.592
2420.	12.616	13.792	2920.	10.209	11.541
2430.	12.575	13.749	2930.	10.152	11.490
2440.	12.533	13.706	2940.	10.094	11.438
2450.	12.492	13.664	2950.	10.035	11.386
2460.	12.450	13.621	2960.	9.976	11.333
2470.	12.408	13.578	2970.	9.917	11.280
2480.	12.365	13.534	2980.	9.857	11.226
2490.	12.322	13.491	2990.	9.796	11.171

Table 8 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
3000.	9.735	11.116	3500.	6.383	7.849
3010.	9.673	11.060	3510.	6.328	7.789
3020.	9.610	11.004	3520.	6.273	7.730
3030.	9.547	10.947	3530.	6.219	7.672
3040.	9.484	10.889	3540.	6.167	7.615
3050.	9.420	10.831	3550.	6.115	7.558
3060.	9.355	10.772	3560.	6.064	7.502
3070.	9.290	10.712	3570.	6.014	7.447
3080.	9.224	10.651	3580.	5.965	7.393
3090.	9.158	10.590	3590.	5.917	7.339
3100.	9.091	10.528	3600.	5.869	7.286
3110.	9.023	10.466	3610.	5.822	7.234
3120.	8.955	10.402	3620.	5.777	7.182
3130.	8.886	10.338	3630.	5.731	7.131
3140.	8.817	10.274	3640.	5.687	7.080
3150.	8.748	10.209	3650.	5.643	7.030
3160.	8.678	10.143	3660.	5.600	6.981
3170.	8.608	10.076	3670.	5.558	6.932
3180.	8.537	10.009	3680.	5.516	6.883
3190.	8.466	9.942	3690.	5.474	6.835
3200.	8.395	9.874	3700.	5.433	6.788
3210.	8.323	9.805	3710.	5.393	6.741
3220.	8.252	9.737	3720.	5.353	6.694
3230.	8.180	9.667	3730.	5.314	6.648
3240.	8.108	9.598	3740.	5.275	6.602
3250.	8.036	9.528	3750.	5.236	6.557
3260.	7.964	9.458	3760.	5.198	6.511
3270.	7.892	9.388	3770.	5.160	6.466
3280.	7.820	9.317	3780.	5.122	6.422
3290.	7.749	9.247	3790.	5.084	6.378
3300.	7.678	9.177	3800.	5.047	6.333
3310.	7.607	9.106	3810.	5.010	6.290
3320.	7.536	9.036	3820.	4.973	6.246
3330.	7.466	8.966	3830.	4.937	6.202
3340.	7.396	8.896	3840.	4.900	6.159
3350.	7.327	8.827	3850.	4.864	6.116
3360.	7.258	8.758	3860.	4.827	6.073
3370.	7.191	8.689	3870.	4.791	6.030
3380.	7.123	8.621	3880.	4.755	5.988
3390.	7.057	8.553	3890.	4.719	5.945
3400.	6.991	8.486	3900.	4.683	5.902
3410.	6.926	8.419	3910.	4.646	5.860
3420.	6.862	8.353	3920.	4.610	5.817
3430.	6.799	8.287	3930.	4.574	5.775
3440.	6.737	8.223	3940.	4.538	5.733
3450.	6.676	8.159	3950.	4.502	5.690
3460.	6.615	8.095	3960.	4.465	5.648
3470.	6.556	8.032	3970.	4.429	5.606
3480.	6.497	7.971	3980.	4.392	5.564
3490.	6.440	7.909	3990.	4.356	5.521

Table 8 continued

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
4000.	4.319	5.479	4500.	2.271	3.287
4010.	4.282	5.437	4510.	2.227	3.241
4020.	4.245	5.395	4520.	2.182	3.196
4030.	4.208	5.352	4530.	2.138	3.151
4040.	4.171	5.310	4540.	2.093	3.105
4050.	4.133	5.267	4550.	2.049	3.060
4060.	4.095	5.225	4560.	2.004	3.014
4070.	4.058	5.182	4570.	1.959	2.969
4080.	4.020	5.140	4580.	1.914	2.923
4090.	3.982	5.097	4590.	1.869	2.878
4100.	3.943	5.054	4600.	1.824	2.832
4110.	3.905	5.011	4610.	1.779	2.787
4120.	3.866	4.969	4620.	1.734	2.741
4130.	3.827	4.926	4630.	1.689	2.695
4140.	3.788	4.883	4640.	1.643	2.650
4150.	3.749	4.839	4650.	1.598	2.604
4160.	3.709	4.796	4660.	1.553	2.558
4170.	3.670	4.753	4670.	1.508	2.513
4180.	3.630	4.710	4680.	1.462	2.467
4190.	3.590	4.666	4690.	1.417	2.421
4200.	3.550	4.623	4700.	1.371	2.376
4210.	3.509	4.579	4710.	1.326	2.330
4220.	3.469	4.536	4720.	1.280	2.284
4230.	3.428	4.492	4730.	1.235	2.238
4240.	3.387	4.448	4740.	1.189	2.193
4250.	3.346	4.404	4750.	1.144	2.147
4260.	3.305	4.360	4760.	1.098	2.101
4270.	3.263	4.316	4770.	1.053	2.055
4280.	3.222	4.272	4780.	1.007	2.009
4290.	3.180	4.228	4790.	0.961	1.964
4300.	3.138	4.184	4800.	0.916	1.918
4310.	3.096	4.139	4810.	0.870	1.872
4320.	3.054	4.095	4820.	0.824	1.826
4330.	3.011	4.051	4830.	0.779	1.780
4340.	2.969	4.006	4840.	0.733	1.734
4350.	2.926	3.962	4850.	0.687	1.689
4360.	2.883	3.917	4860.	0.641	1.643
4370.	2.840	3.872	4870.	0.596	1.597
4380.	2.797	3.827	4880.	0.550	1.551
4390.	2.754	3.783	4890.	0.504	1.505
4400.	2.711	3.738	4900.	0.458	1.459
4410.	2.667	3.693	4910.	0.412	1.413
4420.	2.624	3.648	4920.	0.367	1.367
4430.	2.580	3.603	4930.	0.321	1.322
4440.	2.536	3.558	4940.	0.275	1.276
4450.	2.492	3.513	4950.	0.229	1.230
4460.	2.448	3.468	4960.	0.183	1.184
4470.	2.404	3.423	4970.	0.138	1.138
4480.	2.360	3.377	4980.	0.092	1.092
4490.	2.316	3.332	4990.	0.046	1.046
			5000.	0.000	1.000

Table 8 continued

x/m	bed lev/m	stage/m
0.	5.152	6.152
10.	5.106	6.107
20.	5.058	6.061
30.	5.008	6.015
40.	4.958	5.969
50.	4.906	5.924
60.	4.853	5.879
70.	4.800	5.834
80.	4.745	5.789
90.	4.690	5.744
100.	4.635	5.700
110.	4.578	5.655
120.	4.521	5.611
130.	4.464	5.566
140.	4.406	5.522
150.	4.347	5.477
160.	4.288	5.432
170.	4.228	5.387
180.	4.168	5.342
190.	4.107	5.296
200.	4.045	5.250
210.	3.983	5.203
220.	3.921	5.156
230.	3.857	5.108
240.	3.793	5.059
250.	3.728	5.009
260.	3.663	4.959
270.	3.596	4.907
280.	3.529	4.854
290.	3.461	4.800
300.	3.392	4.745
310.	3.323	4.689
320.	3.252	4.631
330.	3.181	4.572
340.	3.109	4.512
350.	3.036	4.450
360.	2.962	4.387
370.	2.888	4.323
380.	2.813	4.257
390.	2.738	4.190
400.	2.662	4.123
410.	2.586	4.054
420.	2.510	3.984
430.	2.434	3.914
440.	2.358	3.843
450.	2.282	3.772
460.	2.207	3.701
470.	2.133	3.630
480.	2.060	3.558
490.	1.988	3.488
500.	1.918	3.418
510.	1.849	3.348
520.	1.781	3.280
530.	1.716	3.213
540.	1.653	3.146
550.	1.592	3.082
560.	1.533	3.018
570.	1.476	2.957
580.	1.421	2.896
590.	1.369	2.837
600.	1.319	2.780
610.	1.272	2.724
620.	1.226	2.670
630.	1.183	2.618
640.	1.142	2.566
650.	1.102	2.516
660.	1.065	2.467
670.	1.029	2.420
680.	0.995	2.373
690.	0.962	2.328
700.	0.930	2.283
710.	0.900	2.239
720.	0.871	2.196
730.	0.843	2.154
740.	0.815	2.112
750.	0.789	2.070
760.	0.763	2.029
770.	0.737	1.988
780.	0.711	1.947
790.	0.686	1.906
800.	0.661	1.866
810.	0.636	1.825
820.	0.610	1.784
830.	0.584	1.743
840.	0.558	1.702
850.	0.531	1.661
860.	0.503	1.619
870.	0.475	1.577
880.	0.446	1.535
890.	0.416	1.492
900.	0.385	1.449
910.	0.352	1.406
920.	0.319	1.362
930.	0.284	1.318
940.	0.248	1.273
950.	0.210	1.228
960.	0.171	1.183
970.	0.131	1.137
980.	0.089	1.092
990.	0.045	1.046
1000.	0.000	1.000

Table 9: Bed level and stage data for VR2

x/m	bed lev/m	stage/m
0.	4.854	5.496
10.	4.777	5.428
20.	4.702	5.362
30.	4.629	5.297
40.	4.556	5.234
50.	4.485	5.172
60.	4.414	5.110
70.	4.345	5.050
80.	4.276	4.990
90.	4.208	4.931
100.	4.140	4.872
110.	4.072	4.814
120.	4.005	4.756
130.	3.938	4.698
140.	3.871	4.640
150.	3.804	4.583
160.	3.737	4.525
170.	3.670	4.466
180.	3.603	4.408
190.	3.536	4.349
200.	3.468	4.290
210.	3.399	4.230
220.	3.331	4.170
230.	3.262	4.109
240.	3.192	4.048
250.	3.122	3.986
260.	3.051	3.923
270.	2.980	3.859
280.	2.908	3.795
290.	2.836	3.730
300.	2.763	3.664
310.	2.689	3.597
320.	2.615	3.529
330.	2.540	3.461
340.	2.465	3.391
350.	2.389	3.321
360.	2.313	3.251
370.	2.237	3.179
380.	2.161	3.108
390.	2.084	3.035
400.	2.007	2.962
410.	1.930	2.889
420.	1.854	2.816
430.	1.778	2.743
440.	1.702	2.670
450.	1.627	2.597
460.	1.552	2.524
470.	1.478	2.451
480.	1.405	2.379
490.	1.333	2.308
500.	1.262	2.237
510.	1.199	2.393
520.	1.158	2.368
530.	1.128	2.323
540.	1.100	2.269
550.	1.071	2.211
560.	1.038	2.152
570.	1.002	2.095
580.	0.964	2.040
590.	0.924	1.988
600.	0.885	1.939
610.	0.846	1.893
620.	0.807	1.851
630.	0.770	1.811
640.	0.734	1.773
650.	0.699	1.738
660.	0.665	1.705
670.	0.633	1.674
680.	0.603	1.645
690.	0.573	1.617
700.	0.545	1.591
710.	0.518	1.566
720.	0.493	1.543
730.	0.468	1.521
740.	0.445	1.499
750.	0.422	1.479
760.	0.400	1.460
770.	0.379	1.442
780.	0.359	1.424
790.	0.340	1.407
800.	0.321	1.391
810.	0.302	1.375
820.	0.284	1.360
830.	0.267	1.345
840.	0.250	1.331
850.	0.233	1.317
860.	0.217	1.303
870.	0.201	1.290
880.	0.185	1.277
890.	0.170	1.264
900.	0.154	1.251
910.	0.139	1.239
920.	0.124	1.226
930.	0.108	1.214
940.	0.093	1.202
950.	0.078	1.189
960.	0.063	1.177
970.	0.047	1.164
980.	0.032	1.151
990.	0.016	1.138
1000.	0.000	1.125

Table 10: Bed level and stage data for VR3

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	3.484	4.392	510.	1.399	2.748
10.	3.414	4.324	520.	1.392	2.740
20.	3.345	4.256	530.	1.387	2.731
30.	3.276	4.189	540.	1.383	2.722
40.	3.207	4.123	550.	1.380	2.712
50.	3.140	4.057	560.	1.378	2.703
60.	3.073	3.993	570.	1.377	2.693
70.	3.007	3.930	580.	1.376	2.682
80.	2.941	3.868	590.	1.375	2.671
90.	2.877	3.808	600.	1.375	2.659
100.	2.814	3.749	610.	1.375	2.646
110.	2.752	3.691	620.	1.375	2.632
120.	2.691	3.636	630.	1.374	2.617
130.	2.631	3.582	640.	1.373	2.602
140.	2.573	3.530	650.	1.371	2.585
150.	2.516	3.480	660.	1.368	2.567
160.	2.461	3.432	670.	1.364	2.548
170.	2.407	3.386	680.	1.359	2.527
180.	2.355	3.342	690.	1.352	2.504
190.	2.304	3.300	700.	1.343	2.480
200.	2.255	3.261	710.	1.332	2.454
210.	2.207	3.224	720.	1.319	2.426
220.	2.161	3.189	730.	1.304	2.397
230.	2.116	3.156	740.	1.286	2.365
240.	2.073	3.126	750.	1.266	2.331
250.	2.031	3.097	760.	1.243	2.295
260.	1.991	3.070	770.	1.217	2.257
270.	1.952	3.045	780.	1.189	2.217
280.	1.915	3.022	790.	1.158	2.175
290.	1.878	3.001	800.	1.124	2.130
300.	1.843	2.981	810.	1.087	2.084
310.	1.810	2.962	820.	1.048	2.035
320.	1.777	2.945	830.	1.006	1.984
330.	1.746	2.929	840.	0.961	1.932
340.	1.716	2.914	850.	0.914	1.877
350.	1.686	2.900	860.	0.865	1.821
360.	1.659	2.887	870.	0.813	1.763
370.	1.632	2.875	880.	0.759	1.704
380.	1.607	2.864	890.	0.704	1.643
390.	1.582	2.853	900.	0.646	1.581
400.	1.559	2.843	910.	0.587	1.518
410.	1.538	2.833	920.	0.526	1.453
420.	1.518	2.824	930.	0.464	1.388
430.	1.499	2.815	940.	0.401	1.321
440.	1.481	2.806	950.	0.336	1.254
450.	1.465	2.798	960.	0.271	1.186
460.	1.451	2.789	970.	0.204	1.117
470.	1.437	2.781	980.	0.137	1.048
480.	1.426	2.773	990.	0.069	0.978
490.	1.415	2.765	1000.	0.000	0.908
500.	1.406	2.756			

Table 11: Bed level and stage data for UE1

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0.	0.170	1.683	510.	-0.642	1.606
10.	0.163	1.679	520.	-0.639	1.606
20.	0.157	1.676	530.	-0.633	1.606
30.	0.150	1.672	540.	-0.625	1.606
40.	0.143	1.668	550.	-0.615	1.606
50.	0.135	1.664	560.	-0.603	1.605
60.	0.127	1.661	570.	-0.588	1.605
70.	0.118	1.657	580.	-0.572	1.605
80.	0.109	1.654	590.	-0.554	1.604
90.	0.100	1.651	600.	-0.535	1.604
100.	0.089	1.647	610.	-0.514	1.604
110.	0.079	1.644	620.	-0.492	1.603
120.	0.067	1.641	630.	-0.469	1.603
130.	0.055	1.639	640.	-0.446	1.602
140.	0.042	1.636	650.	-0.421	1.602
150.	0.028	1.634	660.	-0.397	1.601
160.	0.013	1.631	670.	-0.372	1.601
170.	-0.002	1.629	680.	-0.347	1.600
180.	-0.019	1.627	690.	-0.322	1.599
190.	-0.036	1.625	700.	-0.297	1.598
200.	-0.054	1.623	710.	-0.273	1.597
210.	-0.074	1.622	720.	-0.250	1.596
220.	-0.094	1.620	730.	-0.227	1.595
230.	-0.115	1.619	740.	-0.205	1.593
240.	-0.137	1.618	750.	-0.184	1.592
250.	-0.159	1.616	760.	-0.164	1.590
260.	-0.183	1.615	770.	-0.145	1.588
270.	-0.207	1.615	780.	-0.128	1.586
280.	-0.232	1.614	790.	-0.111	1.584
290.	-0.257	1.613	800.	-0.096	1.582
300.	-0.283	1.612	810.	-0.081	1.580
310.	-0.309	1.612	820.	-0.068	1.577
320.	-0.335	1.611	830.	-0.057	1.575
330.	-0.362	1.611	840.	-0.046	1.572
340.	-0.388	1.610	850.	-0.037	1.569
350.	-0.413	1.610	860.	-0.028	1.566
360.	-0.438	1.610	870.	-0.021	1.563
370.	-0.463	1.609	880.	-0.015	1.559
380.	-0.487	1.609	890.	-0.010	1.556
390.	-0.509	1.609	900.	-0.006	1.552
400.	-0.531	1.609	910.	-0.002	1.549
410.	-0.551	1.608	920.	0.001	1.545
420.	-0.569	1.608	930.	0.003	1.541
430.	-0.586	1.608	940.	0.004	1.538
440.	-0.600	1.608	950.	0.004	1.534
450.	-0.613	1.607	960.	0.004	1.530
460.	-0.624	1.607	970.	0.004	1.526
470.	-0.632	1.607	980.	0.003	1.522
480.	-0.638	1.607	990.	0.002	1.518
490.	-0.642	1.607	1000.	0.000	1.514
500.	-0.643	1.607			

Table 12: Bed level and stage data for UE2

x/m	bed lev/m	stage/m
0.	5.560	6.210
10.	5.429	6.083
20.	5.300	5.958
30.	5.174	5.836
40.	5.051	5.716
50.	4.930	5.599
60.	4.812	5.483
70.	4.695	5.370
80.	4.581	5.258
90.	4.469	5.149
100.	4.358	5.041
110.	4.249	4.934
120.	4.141	4.830
130.	4.036	4.726
140.	3.931	4.624
150.	3.828	4.523
160.	3.726	4.424
170.	3.626	4.325
180.	3.526	4.228
190.	3.428	4.131
200.	3.331	4.036
210.	3.234	3.941
220.	3.139	3.848
230.	3.045	3.755
240.	2.951	3.663
250.	2.858	3.572
260.	2.767	3.481
270.	2.675	3.391
280.	2.585	3.302
290.	2.495	3.214
300.	2.406	3.126
310.	2.317	3.038
320.	2.229	2.951
330.	2.142	2.865
340.	2.055	2.779
350.	1.968	2.694
360.	1.882	2.609
370.	1.797	2.524
380.	1.712	2.440
390.	1.627	2.356
400.	1.543	2.273
410.	1.459	2.190
420.	1.376	2.107
430.	1.293	2.025
440.	1.210	1.943
450.	1.128	1.861
460.	1.045	1.780
470.	0.964	1.698
480.	0.882	1.617
490.	0.801	1.537
500.	0.720	1.456
510.	0.647	1.627
520.	0.588	1.631
530.	0.535	1.631
540.	0.488	1.630
550.	0.447	1.628
560.	0.413	1.625
570.	0.385	1.622
580.	0.361	1.618
590.	0.342	1.614
600.	0.326	1.610
610.	0.312	1.607
620.	0.299	1.603
630.	0.288	1.599
640.	0.278	1.595
650.	0.268	1.591
660.	0.259	1.588
670.	0.250	1.584
680.	0.242	1.581
690.	0.234	1.577
700.	0.226	1.574
710.	0.217	1.570
720.	0.210	1.567
730.	0.202	1.564
740.	0.194	1.561
750.	0.186	1.558
760.	0.178	1.555
770.	0.171	1.552
780.	0.163	1.549
790.	0.155	1.546
800.	0.148	1.543
810.	0.140	1.541
820.	0.133	1.538
830.	0.125	1.536
840.	0.118	1.533
850.	0.110	1.531
860.	0.103	1.528
870.	0.096	1.526
880.	0.088	1.524
890.	0.081	1.521
900.	0.073	1.519
910.	0.066	1.517
920.	0.059	1.515
930.	0.051	1.513
940.	0.044	1.511
950.	0.037	1.509
960.	0.029	1.507
970.	0.022	1.505
980.	0.015	1.503
990.	0.007	1.502
1000.	0.000	1.500

Table 13: Bed level and stage data for UE3

x/m	bed lev/m	stage/m
0	6.587	7.328
10	6.535	7.276
20	6.482	7.224
30	6.430	7.172
40	6.378	7.120
50	6.326	7.068
60	6.274	7.015
70	6.222	6.963
80	6.170	6.911
90	6.117	6.859
100	6.065	6.806
110	6.013	6.754
120	5.961	6.702
130	5.908	6.649
140	5.856	6.596
150	5.804	6.543
160	5.751	6.490
170	5.698	6.437
180	5.645	6.383
190	5.592	6.329
200	5.539	6.275
210	5.485	6.220
220	5.431	6.164
230	5.377	6.108
240	5.322	6.051
250	5.267	5.993
260	5.210	5.933
270	5.153	5.873
280	5.095	5.810
290	5.035	5.747
300	4.975	5.681
310	4.912	5.613
320	4.848	5.543
330	4.782	5.471
340	4.713	5.396
350	4.642	5.318
360	4.569	5.237
370	4.492	5.153
380	4.412	5.065
390	4.329	4.975
400	4.243	4.881
410	4.153	4.783
420	4.059	4.683
430	3.963	4.580
440	3.864	4.475
450	3.762	4.368
460	3.658	4.260
470	3.552	4.150
480	3.446	4.041
490	3.339	3.933
500	3.233	3.826

x/m	bed lev/m	stage/m
510	3.128	3.722
520	3.024	3.620
530	2.923	3.521
540	2.823	3.425
550	2.727	3.333
560	2.634	3.245
570	2.543	3.161
580	2.456	3.080
590	2.372	3.003
600	2.291	2.929
610	2.213	2.858
620	2.137	2.790
630	2.064	2.725
640	1.993	2.662
650	1.924	2.600
660	1.858	2.540
670	1.793	2.482
680	1.729	2.425
690	1.667	2.369
700	1.607	2.313
710	1.547	2.259
720	1.489	2.205
730	1.432	2.151
740	1.375	2.098
750	1.319	2.045
760	1.264	1.993
770	1.209	1.940
780	1.155	1.888
790	1.101	1.835
800	1.048	1.783
810	0.994	1.731
820	0.941	1.679
830	0.888	1.627
840	0.836	1.575
850	0.783	1.523
860	0.731	1.471
870	0.678	1.419
880	0.626	1.367
890	0.574	1.315
900	0.521	1.263
910	0.469	1.211
920	0.417	1.158
930	0.365	1.106
940	0.313	1.054
950	0.261	1.002
960	0.208	0.950
970	0.156	0.898
980	0.104	0.846
990	0.052	0.794
1000	0.000	0.742

Table 14: Bed level and stage data for UR2

x/m	bed lev/m	stage/m
0	12.208	13.224
50	11.989	13.008
100	11.771	12.793
150	11.556	12.581
200	11.342	12.370
250	11.131	12.162
300	10.921	11.956
350	10.714	11.752
400	10.508	11.550
450	10.305	11.351
500	10.104	11.153
550	9.905	10.958
600	9.708	10.764
650	9.513	10.573
700	9.320	10.385
750	9.130	10.198
800	8.941	10.014
850	8.755	9.831
900	8.570	9.651
950	8.388	9.474
1000	8.208	9.298
1050	8.031	9.125
1100	7.855	8.954
1150	7.682	8.785
1200	7.510	8.619
1250	7.341	8.455
1300	7.174	8.293
1350	7.010	8.133
1400	6.847	7.976
1450	6.687	7.821
1500	6.529	7.668
1550	6.373	7.518
1600	6.219	7.369
1650	6.067	7.224
1700	5.917	7.080
1750	5.770	6.939
1800	5.625	6.799
1850	5.482	6.663
1900	5.341	6.528
1950	5.202	6.396
2000	5.065	6.266
2050	4.931	6.138
2100	4.798	6.012
2150	4.668	5.889
2200	4.540	5.768
2250	4.414	5.649
2300	4.289	5.532
2350	4.167	5.418
2400	4.047	5.306
2450	3.929	5.195
2500	3.813	5.087
2550	3.699	4.981
2600	3.587	4.878
2650	3.476	4.776
2700	3.368	4.677
2750	3.262	4.579
2800	3.157	4.484
2850	3.054	4.390
2900	2.953	4.299
2950	2.854	4.209
3000	2.757	4.122
3050	2.661	4.037
3100	2.568	3.953
3150	2.476	3.872
3200	2.385	3.792
3250	2.296	3.714
3300	2.209	3.638
3350	2.124	3.564
3400	2.040	3.492
3450	1.958	3.421
3500	1.877	3.352
3550	1.797	3.285
3600	1.719	3.220
3650	1.643	3.156
3700	1.568	3.094
3750	1.494	3.034
3800	1.422	2.975
3850	1.351	2.918
3900	1.281	2.863
3950	1.212	2.808
4000	1.145	2.756
4050	1.079	2.705
4100	1.014	2.655
4150	0.950	2.607
4200	0.887	2.560
4250	0.825	2.514
4300	0.764	2.470
4350	0.704	2.427
4400	0.646	2.386
4450	0.588	2.346
4500	0.531	2.307
4550	0.474	2.269
4600	0.419	2.232
4650	0.364	2.196
4700	0.310	2.162
4750	0.257	2.129
4800	0.204	2.096
4850	0.152	2.065
4900	0.101	2.035
4950	0.050	2.006
5000	0.000	1.978

Table 15: Bed level and stage data for UR4

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	5.630	6.596	510	4.081	4.815
10	5.608	6.572	520	4.026	4.753
20	5.586	6.549	530	3.969	4.689
30	5.564	6.525	540	3.910	4.623
40	5.542	6.501	550	3.849	4.555
50	5.519	6.477	560	3.786	4.485
60	5.497	6.453	570	3.721	4.413
70	5.474	6.428	580	3.654	4.340
80	5.452	6.404	590	3.585	4.265
90	5.429	6.379	600	3.514	4.189
100	5.406	6.354	610	3.441	4.111
110	5.383	6.328	620	3.366	4.032
120	5.360	6.303	630	3.290	3.951
130	5.336	6.277	640	3.213	3.870
140	5.313	6.251	650	3.134	3.787
150	5.289	6.224	660	3.054	3.703
160	5.266	6.197	670	2.972	3.618
170	5.242	6.170	680	2.889	3.533
180	5.217	6.143	690	2.806	3.447
190	5.193	6.115	700	2.721	3.360
200	5.168	6.087	710	2.636	3.272
210	5.143	6.058	720	2.549	3.184
220	5.118	6.029	730	2.463	3.095
230	5.092	5.999	740	2.375	3.006
240	5.066	5.969	750	2.287	2.917
250	5.040	5.938	760	2.198	2.827
260	5.013	5.907	770	2.109	2.736
270	4.986	5.875	780	2.020	2.646
280	4.958	5.843	790	1.930	2.555
290	4.930	5.810	800	1.839	2.464
300	4.901	5.776	810	1.749	2.373
310	4.872	5.741	820	1.658	2.281
320	4.842	5.706	830	1.567	2.190
330	4.812	5.669	840	1.476	2.098
340	4.780	5.632	850	1.384	2.006
350	4.748	5.594	860	1.293	1.914
360	4.715	5.555	870	1.201	1.822
370	4.682	5.515	880	1.109	1.730
380	4.647	5.474	890	1.017	1.637
390	4.611	5.432	900	0.925	1.545
400	4.575	5.388	910	0.833	1.453
410	4.537	5.344	920	0.740	1.360
420	4.498	5.298	930	0.648	1.267
430	4.458	5.250	940	0.556	1.175
440	4.416	5.202	950	0.463	1.082
450	4.373	5.151	960	0.371	0.990
460	4.329	5.100	970	0.278	0.897
470	4.282	5.046	980	0.185	0.804
480	4.235	4.991	990	0.093	0.711
490	4.185	4.934	1000	0.000	0.619
500	4.134	4.876			

Table 16: Bed level and stage data for UR6

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	5.579	6.123	510	0.782	1.726
10	5.449	5.998	520	0.724	1.732
20	5.321	5.875	530	0.674	1.732
30	5.197	5.755	540	0.628	1.730
40	5.075	5.637	550	0.583	1.729
50	4.956	5.522	560	0.539	1.727
60	4.840	5.410	570	0.497	1.724
70	4.725	5.299	580	0.458	1.722
80	4.613	5.191	590	0.423	1.718
90	4.503	5.084	600	0.390	1.715
100	4.394	4.979	610	0.361	1.711
110	4.287	4.875	620	0.335	1.706
120	4.182	4.773	630	0.312	1.702
130	4.079	4.673	640	0.291	1.697
140	3.977	4.574	650	0.272	1.692
150	3.876	4.476	660	0.256	1.686
160	3.777	4.379	670	0.241	1.681
170	3.679	4.284	680	0.228	1.675
180	3.582	4.189	690	0.216	1.670
190	3.486	4.096	700	0.205	1.664
200	3.392	4.004	710	0.195	1.658
210	3.298	3.912	720	0.186	1.652
220	3.205	3.821	730	0.177	1.646
230	3.113	3.732	740	0.168	1.641
240	3.022	3.642	750	0.160	1.635
250	2.932	3.554	760	0.153	1.629
260	2.843	3.466	770	0.145	1.623
270	2.754	3.379	780	0.138	1.617
280	2.666	3.293	790	0.131	1.611
290	2.579	3.207	800	0.125	1.605
300	2.492	3.122	810	0.118	1.599
310	2.406	3.037	820	0.111	1.592
320	2.320	2.953	830	0.105	1.586
330	2.235	2.870	840	0.099	1.580
340	2.151	2.787	850	0.092	1.574
350	2.067	2.704	860	0.086	1.568
360	1.983	2.622	870	0.080	1.562
370	1.900	2.540	880	0.074	1.556
380	1.818	2.458	890	0.067	1.550
390	1.736	2.377	900	0.061	1.544
400	1.654	2.296	910	0.055	1.538
410	1.572	2.216	920	0.049	1.532
420	1.491	2.136	930	0.043	1.526
430	1.411	2.056	940	0.037	1.520
440	1.330	1.976	950	0.030	1.514
450	1.250	1.897	960	0.024	1.507
460	1.170	1.818	970	0.018	1.501
470	1.091	1.739	980	0.012	1.495
480	1.012	1.661	990	0.006	1.489
490	0.933	1.583	1000	0.000	1.483
500	0.854	1.695			

Table 17: Bed level and stage data for UR7

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	5.672	6.216	510	0.875	1.817
10	5.541	6.090	520	0.823	1.819
20	5.414	5.967	530	0.785	1.811
30	5.289	5.847	540	0.755	1.800
40	5.168	5.730	550	0.728	1.788
50	5.049	5.615	560	0.703	1.775
60	4.932	5.502	570	0.680	1.762
70	4.818	5.392	580	0.657	1.750
80	4.705	5.283	590	0.635	1.737
90	4.595	5.176	600	0.614	1.725
100	4.487	5.071	610	0.594	1.713
110	4.380	4.968	620	0.574	1.701
120	4.275	4.866	630	0.555	1.689
130	4.171	4.765	640	0.536	1.677
140	4.069	4.666	650	0.518	1.666
150	3.969	4.568	660	0.500	1.654
160	3.870	4.472	670	0.483	1.643
170	3.772	4.376	680	0.466	1.631
180	3.675	4.282	690	0.449	1.620
190	3.579	4.189	700	0.433	1.609
200	3.484	4.096	710	0.416	1.598
210	3.390	4.005	720	0.401	1.587
220	3.298	3.914	730	0.385	1.577
230	3.206	3.824	740	0.369	1.566
240	3.115	3.735	750	0.354	1.556
250	3.025	3.647	760	0.338	1.545
260	2.935	3.559	770	0.323	1.535
270	2.846	3.472	780	0.308	1.525
280	2.758	3.386	790	0.293	1.515
290	2.671	3.300	800	0.279	1.505
300	2.584	3.215	810	0.264	1.496
310	2.498	3.130	820	0.249	1.486
320	2.413	3.046	830	0.235	1.477
330	2.328	2.962	840	0.220	1.467
340	2.243	2.879	850	0.206	1.458
350	2.159	2.796	860	0.192	1.449
360	2.076	2.714	870	0.178	1.440
370	1.993	2.632	880	0.164	1.431
380	1.910	2.551	890	0.150	1.423
390	1.828	2.470	900	0.136	1.414
400	1.746	2.389	910	0.122	1.406
410	1.665	2.308	920	0.108	1.397
420	1.584	2.228	930	0.094	1.389
430	1.503	2.148	940	0.081	1.381
440	1.423	2.069	950	0.067	1.373
450	1.343	1.990	960	0.054	1.365
460	1.263	1.911	970	0.040	1.357
470	1.183	1.832	980	0.027	1.350
480	1.104	1.754	990	0.013	1.342
490	1.025	1.675	1000	0.000	1.335
500	0.947	1.787			

Table 18: Bed level and stage for UR8

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	5.197	6.155	510	2.716	3.353
10	5.177	6.128	520	2.630	3.265
20	5.157	6.101	530	2.543	3.176
30	5.135	6.072	540	2.456	3.087
40	5.114	6.044	550	2.368	2.997
50	5.091	6.014	560	2.279	2.907
60	5.068	5.984	570	2.190	2.817
70	5.044	5.953	580	2.100	2.727
80	5.019	5.922	590	2.010	2.636
90	4.994	5.889	600	1.920	2.793
100	4.968	5.856	610	1.838	2.827
110	4.941	5.822	620	1.782	2.831
120	4.913	5.787	630	1.751	2.821
130	4.884	5.751	640	1.736	2.804
140	4.854	5.714	650	1.730	2.781
150	4.824	5.676	660	1.729	2.754
160	4.792	5.637	670	1.727	2.722
170	4.759	5.597	680	1.723	2.686
180	4.725	5.556	690	1.715	2.647
190	4.690	5.513	700	1.702	2.605
200	4.654	5.470	710	1.684	2.560
210	4.617	5.425	720	1.661	2.512
220	4.578	5.379	730	1.633	2.461
230	4.538	5.332	740	1.600	2.408
240	4.497	5.283	750	1.562	2.353
250	4.454	5.232	760	1.520	2.296
260	4.409	5.181	770	1.475	2.237
270	4.363	5.127	780	1.427	2.177
280	4.316	5.072	790	1.376	2.115
290	4.266	5.015	800	1.322	2.052
300	4.215	4.957	810	1.266	1.989
310	4.162	4.896	820	1.208	1.924
320	4.107	4.834	830	1.148	1.858
330	4.050	4.770	840	1.087	1.792
340	3.991	4.703	850	1.024	1.724
350	3.930	4.635	860	0.960	1.657
360	3.867	4.566	870	0.895	1.589
370	3.802	4.494	880	0.830	1.520
380	3.734	4.421	890	0.763	1.451
390	3.665	4.346	900	0.696	1.381
400	3.594	4.270	910	0.628	1.312
410	3.522	4.192	920	0.560	1.242
420	3.447	4.113	930	0.491	1.171
430	3.371	4.032	940	0.422	1.101
440	3.294	3.950	950	0.352	1.030
450	3.215	3.868	960	0.282	0.959
460	3.134	3.784	970	0.212	0.888
470	3.053	3.699	980	0.142	0.816
480	2.970	3.614	990	0.071	0.745
490	2.887	3.527	1000	0.000	0.673
500	2.802	3.441			

Table 19: Bed level and stage data for UR10

x/m	bed lev/m	stage/m
0	8.190	8.734
10	8.059	8.608
20	7.932	8.485
30	7.808	8.365
40	7.686	8.248
50	7.567	8.133
60	7.450	8.020
70	7.336	7.910
80	7.223	7.801
90	7.113	7.694
100	7.005	7.589
110	6.898	7.486
120	6.793	7.384
130	6.690	7.283
140	6.588	7.184
150	6.487	7.087
160	6.388	6.990
170	6.290	6.894
180	6.193	6.800
190	6.097	6.707
200	6.002	6.614
210	5.908	6.523
220	5.816	6.432
230	5.724	6.342
240	5.633	6.253
250	5.543	6.165
260	5.453	6.077
270	5.364	5.990
280	5.276	5.904
290	5.189	5.818
300	5.102	5.733
310	5.016	5.648
320	4.931	5.564
330	4.846	5.480
340	4.761	5.397
350	4.677	5.314
360	4.594	5.232
370	4.511	5.150
380	4.428	5.069
390	4.346	4.988
400	4.264	4.907
410	4.183	4.826
420	4.102	4.746
430	4.021	4.666
440	3.941	4.587
450	3.861	4.508
460	3.781	4.429
470	3.701	4.350
480	3.622	4.272
490	3.543	4.193
500	3.465	4.305

x/m	bed lev/m	stage/m
510	3.395	4.333
520	3.352	4.328
530	3.330	4.306
540	3.320	4.273
550	3.312	4.231
560	3.300	4.181
570	3.282	4.123
580	3.254	4.058
590	3.216	3.987
600	3.169	3.909
610	3.112	3.827
620	3.047	3.742
630	2.976	3.653
640	2.899	3.563
650	2.818	3.471
660	2.735	3.380
670	2.649	3.288
680	2.562	3.197
690	2.474	3.106
700	2.386	3.017
710	2.299	2.928
720	2.211	2.841
730	2.124	2.755
740	2.038	2.670
750	1.952	2.586
760	1.868	2.502
770	1.784	2.420
780	1.700	2.339
790	1.618	2.258
800	1.536	2.178
810	1.455	2.099
820	1.375	2.020
830	1.295	1.942
840	1.216	1.864
850	1.137	1.787
860	1.059	1.711
870	0.981	1.634
880	0.904	1.558
890	0.827	1.482
900	0.751	1.407
910	0.674	1.332
920	0.599	1.257
930	0.523	1.182
940	0.448	1.107
950	0.373	1.033
960	0.298	0.959
970	0.223	0.885
980	0.149	0.811
990	0.074	0.737
1000	0.000	0.663

Table 20: Bed level and stage data for UR11

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	6.097	6.855	510	2.849	3.975
10	5.995	6.754	520	2.824	3.949
20	5.894	6.655	530	2.801	3.922
30	5.793	6.555	540	2.777	3.894
40	5.694	6.457	550	2.754	3.866
50	5.594	6.360	560	2.732	3.837
60	5.496	6.264	570	2.709	3.807
70	5.399	6.170	580	2.687	3.777
80	5.303	6.076	590	2.664	3.745
90	5.208	5.985	600	2.640	3.711
100	5.115	5.895	610	2.616	3.677
110	5.023	5.807	620	2.591	3.641
120	4.932	5.720	630	2.565	3.603
130	4.843	5.636	640	2.538	3.564
140	4.756	5.554	650	2.509	3.522
150	4.670	5.474	660	2.479	3.479
160	4.587	5.397	670	2.447	3.434
170	4.506	5.322	680	2.413	3.387
180	4.426	5.250	690	2.376	3.338
190	4.349	5.180	700	2.337	3.286
200	4.274	5.114	710	2.296	3.232
210	4.201	5.049	720	2.252	3.176
220	4.130	4.988	730	2.205	3.117
230	4.061	4.929	740	2.155	3.056
240	3.995	4.873	750	2.103	2.992
250	3.930	4.819	760	2.047	2.925
260	3.868	4.768	770	1.988	2.856
270	3.808	4.720	780	1.927	2.785
280	3.749	4.673	790	1.862	2.711
290	3.693	4.629	800	1.795	2.635
300	3.639	4.588	810	1.724	2.556
310	3.586	4.548	820	1.651	2.475
320	3.535	4.510	830	1.575	2.392
330	3.486	4.474	840	1.497	2.307
340	3.439	4.439	850	1.416	2.220
350	3.393	4.406	860	1.333	2.131
360	3.349	4.374	870	1.247	2.040
370	3.306	4.343	880	1.160	1.948
380	3.265	4.314	890	1.070	1.854
390	3.225	4.285	900	0.979	1.759
400	3.186	4.257	910	0.886	1.663
410	3.150	4.230	920	0.792	1.566
420	3.114	4.204	930	0.697	1.467
430	3.080	4.178	940	0.600	1.368
440	3.047	4.152	950	0.502	1.268
450	3.015	4.127	960	0.403	1.167
460	2.985	4.102	970	0.303	1.065
470	2.956	4.077	980	0.203	0.963
480	2.927	4.051	990	0.102	0.861
490	2.900	4.026	1000	0.000	0.758
500	2.874	4.001			

Table 21: Bed level and stage data for UT1

x/m	bed lev/m	stage/m	x/m	bed lev/m	stage/m
0	4.816	5.366	510	0.791	1.670
10	4.705	5.260	520	0.744	1.672
20	4.597	5.156	530	0.702	1.670
30	4.491	5.055	540	0.664	1.666
40	4.388	4.956	550	0.629	1.661
50	4.287	4.860	560	0.598	1.655
60	4.188	4.765	570	0.569	1.649
70	4.091	4.672	580	0.544	1.642
80	3.997	4.581	590	0.521	1.634
90	3.903	4.491	600	0.501	1.626
100	3.812	4.403	610	0.482	1.618
110	3.722	4.316	620	0.465	1.610
120	3.633	4.231	630	0.448	1.602
130	3.546	4.147	640	0.433	1.593
140	3.460	4.064	650	0.418	1.585
150	3.376	3.982	660	0.404	1.577
160	3.292	3.901	670	0.391	1.569
170	3.210	3.821	680	0.377	1.560
180	3.128	3.743	690	0.364	1.552
190	3.048	3.664	700	0.351	1.544
200	2.968	3.587	710	0.338	1.536
210	2.890	3.511	720	0.326	1.529
220	2.812	3.435	730	0.313	1.521
230	2.735	3.360	740	0.301	1.513
240	2.659	3.286	750	0.289	1.506
250	2.583	3.212	760	0.277	1.498
260	2.509	3.139	770	0.264	1.491
270	2.434	3.067	780	0.252	1.484
280	2.361	2.995	790	0.240	1.477
290	2.288	2.924	800	0.229	1.470
300	2.215	2.853	810	0.217	1.463
310	2.143	2.782	820	0.205	1.456
320	2.072	2.712	830	0.193	1.449
330	2.001	2.643	840	0.181	1.443
340	1.931	2.573	850	0.170	1.436
350	1.860	2.505	860	0.158	1.430
360	1.791	2.436	870	0.147	1.423
370	1.722	2.368	880	0.135	1.417
380	1.653	2.300	890	0.124	1.411
390	1.584	2.233	900	0.112	1.405
400	1.516	2.166	910	0.101	1.399
410	1.448	2.099	920	0.090	1.393
420	1.381	2.033	930	0.078	1.388
430	1.314	1.966	940	0.067	1.382
440	1.247	1.900	950	0.056	1.376
450	1.180	1.834	960	0.045	1.371
460	1.114	1.769	970	0.033	1.366
470	1.048	1.704	980	0.022	1.360
480	0.982	1.638	990	0.011	1.355
490	0.916	1.574	1000	0.000	1.350
500	0.851	1.644			

Table 22: Bed level and stage data for UT3

x/m	bed lev/m	stage/m
0	4.393	5.035
10	4.316	4.967
20	4.241	4.901
30	4.168	4.836
40	4.095	4.773
50	4.024	4.710
60	3.953	4.649
70	3.884	4.589
80	3.815	4.529
90	3.747	4.470
100	3.679	4.411
110	3.611	4.353
120	3.544	4.295
130	3.477	4.237
140	3.410	4.179
150	3.343	4.122
160	3.276	4.064
170	3.209	4.005
180	3.142	3.947
190	3.074	3.888
200	3.007	3.829
210	2.938	3.769
220	2.870	3.709
230	2.801	3.648
240	2.731	3.587
250	2.661	3.525
260	2.590	3.462
270	2.519	3.398
280	2.447	3.334
290	2.375	3.269
300	2.302	3.203
310	2.228	3.136
320	2.154	3.068
330	2.079	2.999
340	2.004	2.930
350	1.928	2.860
360	1.852	2.790
370	1.776	2.718
380	1.699	2.646
390	1.623	2.574
400	1.546	2.501
410	1.469	2.428
420	1.393	2.355
430	1.317	2.282
440	1.241	2.209
450	1.165	2.135
460	1.091	2.063
470	1.017	1.990
480	0.944	1.918
490	0.872	1.847
500	0.801	1.914
510	0.737	1.936
520	0.686	1.927
530	0.645	1.910
540	0.608	1.891
550	0.574	1.872
560	0.543	1.854
570	0.514	1.836
580	0.487	1.818
590	0.462	1.802
600	0.439	1.786
610	0.418	1.770
620	0.398	1.756
630	0.379	1.742
640	0.361	1.729
650	0.345	1.717
660	0.329	1.705
670	0.314	1.694
680	0.300	1.684
690	0.287	1.674
700	0.274	1.665
710	0.262	1.657
720	0.250	1.649
730	0.239	1.641
740	0.228	1.634
750	0.218	1.627
760	0.208	1.620
770	0.198	1.614
780	0.189	1.608
790	0.179	1.602
800	0.170	1.597
810	0.161	1.592
820	0.152	1.586
830	0.144	1.581
840	0.135	1.576
850	0.127	1.572
860	0.119	1.567
870	0.110	1.562
880	0.102	1.558
890	0.094	1.553
900	0.085	1.548
910	0.077	1.544
920	0.069	1.539
930	0.061	1.535
940	0.052	1.530
950	0.044	1.525
960	0.035	1.520
970	0.027	1.516
980	0.018	1.510
990	0.009	1.505
1000	0.000	1.500

Table 23: Bed level and stage data for VR4