Computation of the Net Present Value of a Reservoir with Uncertain Data

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Abstract

This paper presents a direct method to determine the uncertainty in pressure, flow and Net Present Value (NPV) of a reservoir, using the time-dependent one phase 2-dimensional reservoir flow equations. The uncertainty in the solution for pressures and ultimately, NPV, is modelled as a probability distribution function. This is derived from probability distribution functions for input parameters such as permeability.

The method involves a perturbation expansion about a mean of the parameters. Coupled equations for second order approximations to the mean and the field covariance of the solution are developed and solved numerically. This method involves only one (albeit complicated) solution of the equations, and contrasts with the more usual Monte-Carlo approach, where many such solutions are required.

Second-order approximations for the statistical uncertainties of the NPV, such as its mean value and variance, can be evaluated quantitatively. These approximations are then used to estimate the risked value of a field for a given development scenario.

1 Introduction

This paper is an extension of previous work, [1], concerning the mathematical and numerical modeling of physical systems where a precise knowledge of the data that characterises the model is not available. In [1] a statistical perturbation analysis is applied to the fluid flow equations to obtain the probability distribution of the pressures in a reservoir with uncertain data. This technique is extended here to the analysis of the risked value of the oil field. The net present value, or NPV, of the field is used to assess risk, and is defined by

$$NPV = \int_0^\infty \|\mathbf{Q}(t)\| e^{-\alpha t} dt, \tag{1}$$

where $\mathbf{Q}(t)$ is the flow of oil at the relevant production well and α is some weighting factor.

In the mathematical modelling of the field in a deterministic case, the flow term $\mathbf{Q}(\mathbf{t})$ may be easily obtained if values for the pressure are known, and or the field flow equations have been solved for at each time-step. For the simple model dealt with in our previous work, the flow can be obtained just by the formula,

$$\mathbf{Q}(t) = -k \cdot \nabla(p), \tag{2}$$

where k is the permeability and p the pressure.

In previous work, [1], we were dealing with cases where uncertainties in the permeabilities caused corresponding uncertainties to propagate though to the numerical solutions for the pressure. We now investigate how these uncertainties propagate to cause uncertainties in the flow and, more importantly, in the NPV.

2 Treatment of Pressure Equations

The earlier study, [1], is restricted to a fairly straightforward two-dimensional model equation (with the implicit assumption that the results obtained may be generalised to the three-dimensional case.) The model equation

$$\gamma \frac{\partial p}{\partial t} - \nabla(k \nabla p) = f(\mathbf{r}, t), \tag{3}$$

is used, where γ is the compressibility, p the pressure, k the permeability, and $f(\mathbf{r}, t)$ is some forcing function.

The uncertainties under consideration are all contained in the permeability, which is assumed to have a mean value function and a permeability autocorrelation function (P.A.F.), defined as

$$\rho(\mathbf{r_1}, \mathbf{r_2}) = \frac{\langle (k(\mathbf{r_1}) - k_0(\mathbf{r_1}))(k(\mathbf{r_2}) - k_0(\mathbf{r_2})) \rangle}{\sigma_k(\mathbf{r_1}) \sigma_k(\mathbf{r_2})}, \tag{4}$$

a function of the two spatial positions in the permeability field, $\mathbf{r_1}$ and $\mathbf{r_2}$.

We assume the statistical uncertainties in the permeability can be written as a perturbation about the mean value field,

$$k(\mathbf{r}) = k_0(\mathbf{r}) + k_1(\mathbf{r}),$$

where

$$k_0(\mathbf{r}) = \langle k(\mathbf{r}) \rangle.$$

Then this form is substituted into the model equation, to allow a system of hierarchical equations to be developed for the pressure solution, p, where,

$$p = p_0 + p_1 + p_2 + \cdots.$$

For the case of an assumed lognormal distribution function for the permeability, the perturbation expansion is determined about the geometric mean, and the hierarchical equations take on the following form when analysed and discretised:

$$\frac{\gamma p_{0\ ij}^{n+1} - \gamma p_{0\ ij}^{n}}{\Delta t} - \nabla_h (k_{ij}^0 \nabla_h p_{0\ ij}^n) = f_{0\ ij}^n, \tag{5}$$

$$\begin{aligned} & \frac{\gamma \langle k_{i'j'}^1 p_{1\ ij}^{n+1} \rangle - \gamma \langle k_{i'j'}^1 p_{1\ ij}^{n} \rangle}{\Delta t} \\ - \quad & \langle k_{i'j'}^1 \nabla_h (k_{ij}^0 \nabla_h p_{1\ ij}^n) \rangle - \langle k_{i'j'}^1 \nabla_h (k_{ij}^1 \nabla_h p_{0\ ij}^n) \rangle = \langle k_{i'j'}^1 f_{1\ ij}^n \rangle, \end{aligned} \tag{6}$$

$$\frac{\gamma \langle p_{2\ ij}^{n+1} \rangle - \gamma \langle p_{2\ ij}^{n} \rangle}{\Delta t} - \nabla_h (k_{ij}^0 \nabla_h \langle p_{2\ ij}^n \rangle) - \langle \nabla_h (k_{ij}^1 \nabla_h p_{1\ ij}^n) \rangle - \nabla_h \langle k_{ij}^2 \rangle \nabla_h p_{0\ ij}^n = 0. \tag{7}$$

With the same discretisation, the equations for the variance-covariance function for the pressure solution can also be written,

$$\begin{aligned} & \frac{\gamma C_{i'j'ij}^{n+1} - \gamma C_{i'j'ij}^{n}}{\Delta t} \\ - \quad & \nabla_h k_{ij}^0 \nabla_h C_{i'j'ij}^n - \nabla_h \langle k^1 p_1 \rangle_{iji'j'}^n \nabla_h p_{0\ ij}^n \\ - \quad & \nabla_h k_{i'j'}^0 \nabla_h C_{iji'j'}^n - \nabla_h \langle k^1 p_1 \rangle_{i'j'ij}^n \nabla_h p_{0\ i'j'}^n = 0. \end{aligned} \tag{8}$$

The quantity p_0 is denoted to be the *deterministic* solution, which is that which would normally be obtained by substituting the mean value fields into the model equations. The higher order term in pressure, p_2 represents the second order correction to the pressure obtained when including the statistical uncertainties in the model. The term $C_{i'j'ij}^n$ represents the pressure covariance function at time-step n for discretised spatial positions (i', j') and (i, j).

3 Fluid Flow

The straightforward conversion equation to obtain the flow from the pressure of a fluid in a porous medium can be obtained in this model using Darcy's law and is given in simplest form

$$\mathbf{Q} = -k \nabla p. \tag{9}$$

By using the previous assumption from [1] that the pressure may be written as a series, we can substitute this, and the perturbation series for the permeability into the equation (9) to give

$$\mathbf{Q} \simeq -\left(k^0 + k^1 + k^2\right) \nabla \left(p_0 + p_1 + p_2\right), \tag{10}$$

where all terms up to and including second order have been retained.

If we now take mean values on either side, we obtain a vector expression for the mean value of the flow,

$$\langle \mathbf{Q} \rangle \simeq -k_0 \nabla p_0 - \langle k_1 \nabla p_1 \rangle - \langle k_2 \rangle \nabla p_0 - k_0 \nabla \langle p_2 \rangle. \tag{11}$$

Also, the covariance of the flow may be written,

$$Cov_q \simeq \langle k_1 k_1 \rangle \left(\nabla p_0\right)^2 + 2k_0 \nabla p_0 \cdot \langle k_1 \nabla p_1 \rangle + (k_0)^2 \langle (\nabla p_1) \cdot (\nabla p_1) \rangle \tag{12}$$

By using the values for pressure already computed with the methods described in [1],we can then proceed to calculate the first two statistical moments for the flow numerically. They only contain statistical information from the pressure terms which is already available from previous considerations. Both these terms can then be used in order to calculate the net present value and its statistical moments up to second order.

It is fairly straightforward, when approaching the problem in a practical way, to approximate equation (9) with a central difference approximation so that it can be written,

$$\mathbf{Q_{ij}} = -k_{ij} \nabla_h p_{ij}. \tag{13}$$

The equation for the mean value of the flow then takes the form

$$\langle \mathbf{Q_{ij}} \rangle \simeq -k_{ij}^0 \nabla_h p_{ij}^0 - \langle k_{ij}^1 \nabla_h p_{ij}^1 \rangle - \langle k_{ij}^2 \rangle \nabla_h p_{ij}^0 - k_{ij}^0 \nabla \langle p_{ij}^2 \rangle, \tag{14}$$

and the equivalent covariance term is,

$$Cov_{q_{ij}} \simeq \langle k_{ij}^1 k_{ij}^1 \rangle \left(\nabla_h p_{ij}^0\right)^2 + 2k_{ij}^0 \nabla_h p_{ij}^0 \cdot \langle k_{ij}^1 \nabla_h p_{ij}^1 \rangle + (k_{ij}^0)^2 \langle (\nabla_h p_{ij}^1) \cdot (\nabla_h p_{ij}^1) \rangle. \tag{15}$$

It is these discretised forms for the statistical moments of the flow that we use when calculating our numerical approximations to the NPV.

4 Net Present Value

To assess the numerical value of the Net Present Value of the systems we are considering, we must first treat it as a time-dependent variable; that is,

$$NPV = \int_0^T \|\mathbf{Q}\| e^{-\alpha t} dt, \tag{16}$$

where $T \to \infty$. The mean value of this term is then calculated quite straightforwardly,

$$\langle NPV \rangle = \int_0^T \|\langle \mathbf{Q_{ij}} \rangle\| e^{-\alpha t} dt, \qquad (17)$$

and an approximation to the second moment may be written as

$$\langle NPV_2 \rangle = \int_0^T \langle (Q_{ij} - \langle Q_{ij} \rangle^2 \rangle e^{-\alpha t} dt. \qquad (18)$$

We are chiefly interested in how the mean value of the NPV compares with the deterministic solution, which is that obtained by operating the numerical process on the mean value of the permeability field,

$$N\tilde{P}V = \int_0^T \|\tilde{\mathbf{Q}}\| e^{-\alpha t} dt, \qquad (19)$$

where,

$$\tilde{\mathbf{Q}} = -k_{ij}^0 \nabla_h p_{ij}^0. \qquad (20)$$

5 Application

We now apply this technique to a specific example of a discretisation.

Examples of the discrete equations for pressure for the case of a simple five-point difference scheme can be found in [1].

6 Results

In this section we present some illustrative samples of results that we have obtained using this method to solve the full statistical problem.

In each case we consider a single Fourier mode as the initial pressure condition in the reservoir, with no flow conditions around the boundary, and zero forcing function. The region under investigation is square with unit length. All lengths and times are normalised for the purposes of this research.

Using a single Fourier mode as the initial condition means that in the case of a homogeneous mean value for the permeability, the solution for the pressure $p(x, y, t)$ to the p.d.e. under consideration, equation (3), may be expressed as the Fourier mode with an exponentially decaying amplitude,

$$p(x, y, t) = e^{-\pi^2 \frac{\langle k \rangle}{\gamma} t} cos(\pi x). \qquad (21)$$

It is fairly trivial to show by substitution that this is a solution to the model equation, satisfying the zero boundary conditions. We choose this test function as it is a straightforward solution whose deterministic behaviour is well-known.

We observe the values for the N.P.V. at the centre of the square region. Figure 6.1(a) shows the various mean values for the N.P.V. with different permeability variances, compared with the deterministic solution. The homogeneous mean value of the permeability is 0.2. In Figure 6.1(b) the corresponding relative variances are shown for the N.P.V., for the same permeability variances.

In Figure 6.2, we show the equivalent plots in the case of a smaller permeability mean. Here, $kmean = 0.1$. In Figure 6.3, we show the plots of mean of N.P.V. for a larger mean permeability field with $kmean = 0.4$.

Figure 6.1(a)

Figure 6.1(b)

Figure 6.2

Figure 6.3

In Figure 6.1(a) we can see that the mean values for the N.P.V.s corresponding to the smaller values of the permeability field seem to converge to a similar order of magnitude, but significantly different value, to the deterministic solution ($var(k) = 0.0$). The value for the case where the covariance of the permeability field is large with respect to its mean seems not to show convergence.

This effect is repeated in Figures 6.2 and 6.3, with significant convergence being shown in Figure 6.3 where the mean of the permeability is always larger than the mean value field.

7 Conclusions

The types of permeability distribution functions we have considered here have been of limited scope. It is hoped to study more complex instances in future work, such as that with anisotropic correlation lengths and spatially varying mean value fields, as studied in [1].

It is very significant that the mean, or second order approximation to the mean, converges to comparable, but certainly different, values for N.P.V. in cases where the permeability variance is small. This is not true in the cases where significant proportions of the realisations of permeability field lie outside the stability region for the numerical scheme. This, of course is broadly in line with results for the pressure solutions found in [1]

The values for the variance of the N.P.V. do not seem to converge whatever the original choice for permeability variance. This may be due to the fact that the integration is being performed over each instantaneous variance and a true approximation for the variance is not being obtained. More results concerning the variance of the N.P.V will be published in the future.

References

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